

**SOLID PROPELLANT ROCKET MOTOR
DESIGN AND TESTING**

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SOLID PROPELLANT ROCKET MOTOR
DESIGN AND TESTING

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ABSTRACT

This thesis examines the theoretical performance of a solid propellant rocket motor which was developed for launching small amateur research rockets. The theoretical results are presented in the form of two limiting cases concerning the behaviour of the two-phase exhaust flow. Actual testing of the motor is performed utilizing a specially designed test rig in order to compare the results. As well, optimization of the motor's performance is investigated.

The theoretical performance is found to be in good agreement with the test results, providing a basis for future design of larger engines.

No significant improvement in overall motor performance as a result of modifications is foreseen.

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CHAPTER 1: INTRODUCTION

This thesis has three primary objectives: to theoretically analyze the operation of a small solid propellant rocket motor; secondly, to conduct actual testing with which to compare the theoretical result; and thirdly, to modify the propellant and motor design for optimization of motor performance.

Secondary objectives are that knowledge of the actual impulse (thrust-time traits) of the motor can be instrumental in the determination of such parameters as acceleration, velocity and altitude of the rocket during actual flights. Finally, the research results generated by this study would be of value to others interested in the field of amateur rocketry.

1.1 ROCKET MOTOR

The rocket motor under examination in this research study is one of a number of rocket motors developed (largely through experimentation) during twelve years of personal amateur rocketry. This motor has been used quite extensively for rocket flights, capable of propelling a 2 kg., 9 cm. diameter rocket to an altitude of 500 metres, with a high degree of reliability (95 percent success rate on a data base of over 50 firings).

1.2 PROPELLANT

The propellant utilized in the rocket motor is one that is employed exclusively by amateur rocket enthusiasts. It is not a high performance propellant and is not used in prof-

essionally designed rockets. Therefore, there was no known data available on the performance characteristics of this propellant. This necessitated a theoretical analysis coupled with experimentation on combustion product analysis, combustion temperature measurements, burn rate measurement and the effects of varying the oxidizer-fuel (O/F) ratio.

1.3 EXPERIMENTATION

Testing of the actual motor was conducted on a specially designed static testing stand. The static testing stand permitted the motor thrust to be recorded continuously throughout the firing, accomplished by converting the thrust to an electric analogue signal which was amplified, digitized by an A/D converter, then stored in a computer for processing.

Optimization of the propellant involved testing the effects upon its performance by varying O/F ratios. The motor optimization consisted of modification to the nozzle design, in order to attempt to increase the nozzle efficiency.

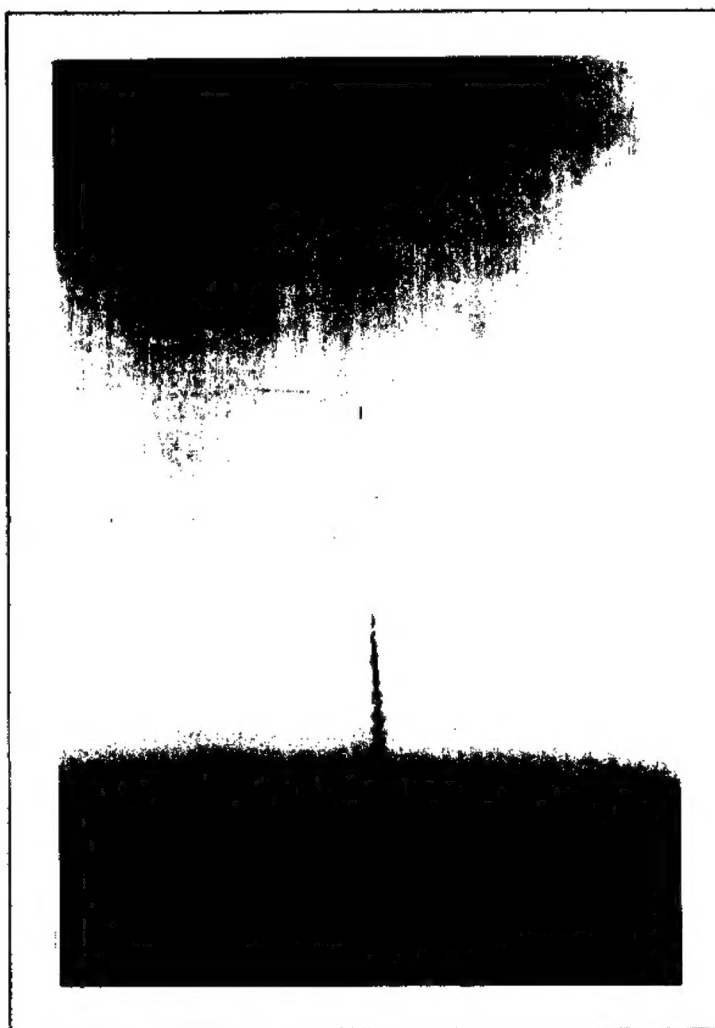


Figure 1.1. Rocket launch equipped with small solid motor.

CHAPTER 2: THEORY

2.1 Solid Propellant Rocket

A typical solid propellant motor (figure 2.1) is of simple construction, containing no movable parts. The major components

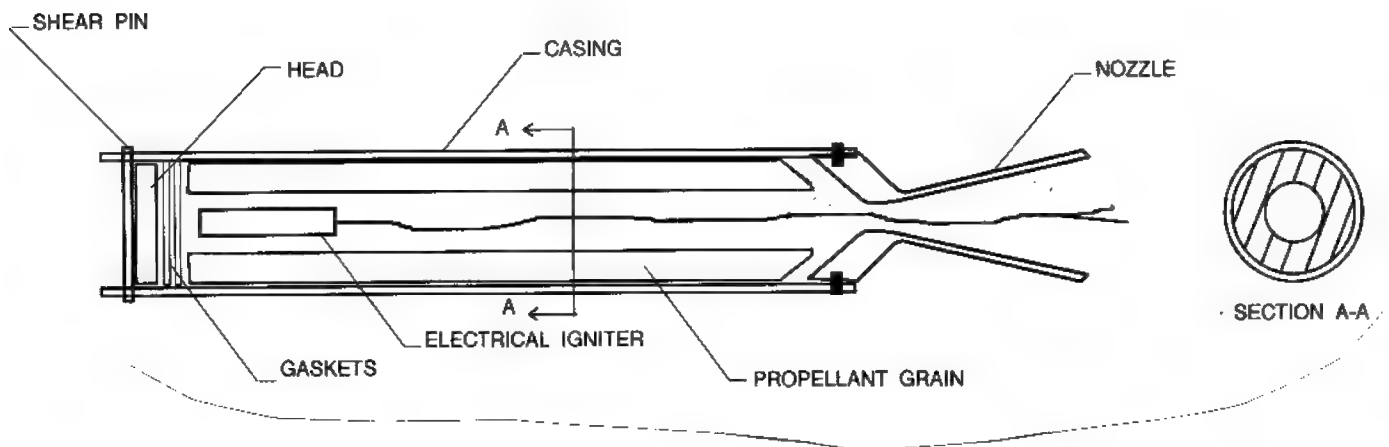


Figure 2.1. A typical solid propellant motor

include the combustion chamber, which contains the propellant grain of particular geometry which in turn determines the surface burning area. Combustion of the grain generates high temperature gases, which are ejected through an exhaust nozzle, designed to accelerate the gases to as high a velocity as possible. The throat is the point in the nozzle of least diameter, and the head being the front of the motor. Combustion is initiated by passing an electrical current through an igniter which contains a charge of black powder.

The theoretical analysis of the rocket motor necessitates certain simplifications, that is, the assumption of an ideal

rocket. The usefulness of this concept is indicated by the fact that measured performance can be expected to be within 1 to 10 per cent of calculated ideal values [2].

An ideal rocket assumes the following:

- (1) The propellant combustion is complete and nonvariant to that assumed by the combustion equation.
- (2) The working substance obeys the perfect gas law.
- (3) There is no friction.
- (4) The combustion and flow in the motor and nozzle is adiabatic.
- (5) Steady-state conditions (unless stated otherwise).
- (6) Expansion of the working fluid occurs in a uniform manner without shock or discontinuities.
- (7) Nozzle flow is one dimensional.
- (8) The gas velocity, pressure, and density is uniform across any cross section normal to the nozzle axis.
- (9) Chemical equilibrium is established in the combustion chamber and does not shift in the nozzle.

Any additional assumptions will be stated as necessary.

2.2 Propellant

2.2.1 Composition

The important considerations for amateur rocket propellants are the availability of the constituents, cost, safety of handling, castability, consistency of performance, and adequate performance. The propellant that is investigated here easily meets or

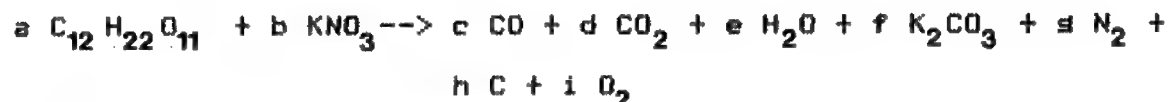
exceeds these requirements.

The propellant consists of sucrose (sugar) fuel with potassium nitrate as the oxidizer. The O/F ratio is chosen on the basis of which ratio gives the greatest overall performance for a given propellant grain design. For most propellants including this one, it is on the fuel rich side of stoichiometric. More accurately, varying the O/F ratio affects both the characteristic exhaust velocity, C_V , (essentially relates to the energy available), the burnrate, r , and the quantity of non-gaseous exhaust products.

These aspects are treated in closer detail elsewhere.

2.2.2 Combustion

The assumed combustion equation is based on the reaction of potassium nitrate (KNO_3) with oxygen (O_2) [3], and upon the decomposition of sucrose ($C_{12}H_{22}O_{11}$) upon heating [4]. The assumed combustion equation is:



where K_2CO_3 represents potassium carbonate, where the coefficients a through i are dependant upon the O/F ratio with c , h or i often being zero.

This equation assumes that secondary exhaust products (such as NO , K_2O , etc.) are formed in negligible quantities. As well, the effects of dissociation are assumed negligible. This second assumption is generally valid for combustion temperatures below 1700 K. particularly at higher pressures [5]. It should be noted that

there are two non-gaseous products formed, K_2CO_3 and carbon, from which no expansion work can be derived.

Knowledge of the combustion equation allows the calculation of the adiabatic flame temperature (AFT), the maximum possible combustion temperature. The AFT is determined from an enthalpy balance:

$$\sum_R n_i \left[\bar{h}_f^\circ + \Delta \bar{h} \right]_i = \sum_P n_e \left[\bar{h}_f^\circ + \Delta \bar{h} \right]_e \quad (2.2)$$

where R and P refer to the reactants and products, respectively,

n_i and n_e are the individual reactant and product molar numbers respectively, $\bar{h}_f^\circ$, the enthalpy of formation per mole, $\Delta \bar{h}$, the standard enthalpy at the specified temperature, per mole, and where:

$$\Delta \bar{h} = \int_{T_1}^{T_K} \bar{C}_P dT + \Delta \bar{h}_{tr}$$

and $\Delta \bar{h}_{tr}$ = enthalpy of transition/mole

T_1 = reference temperature, typically 300 K

T_K = AFT

$\bar{C}_P$ = specific heat at constant pressure/mole

Appendix A contains a sample calculation for the case of O/F = 65/35.

Figure (2.2) shows a comparison of theoretical to actual flame temperatures [7].

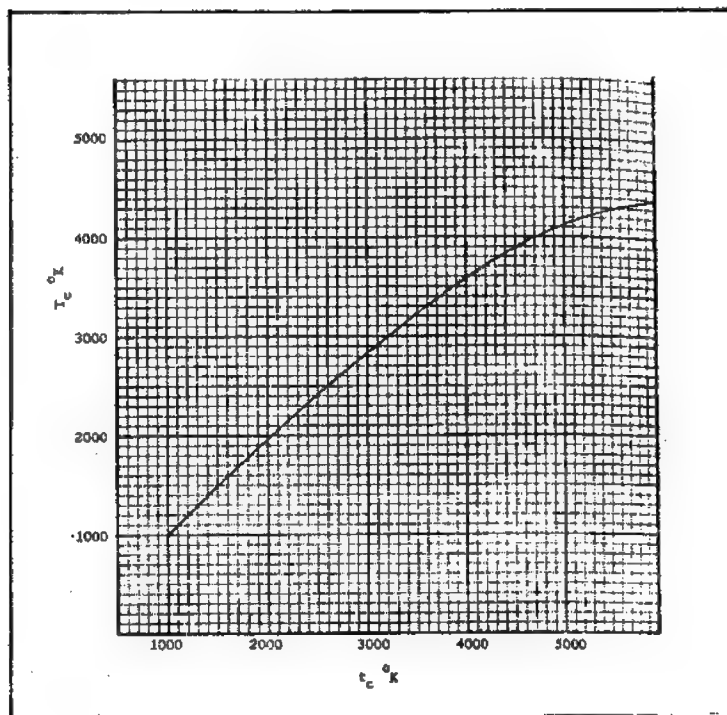
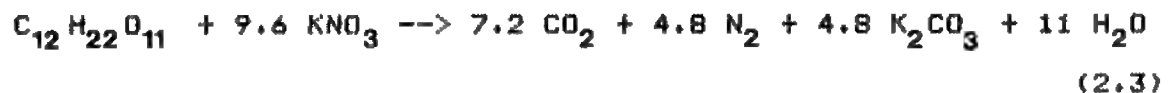


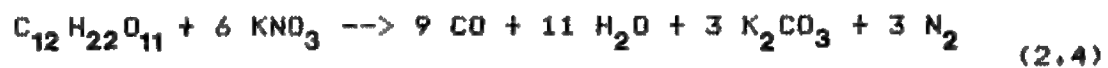
Figure 2.2. The relationship between calculated and actual temperature.

The AFT varies with the O/F ratio, particularly strongly in the range close to stoichiometric, where the AFT is maximum, as shown in figure (2.3).

The two points of sharp slope change are at stoichiometric mixture (peak) where the O/F ratio is 73.3/26.7 and the combustion equation is



and the point where partial oxidation of C to CO is completed, with an O/F ratio of 63.9/36.1, with the combustion equation:



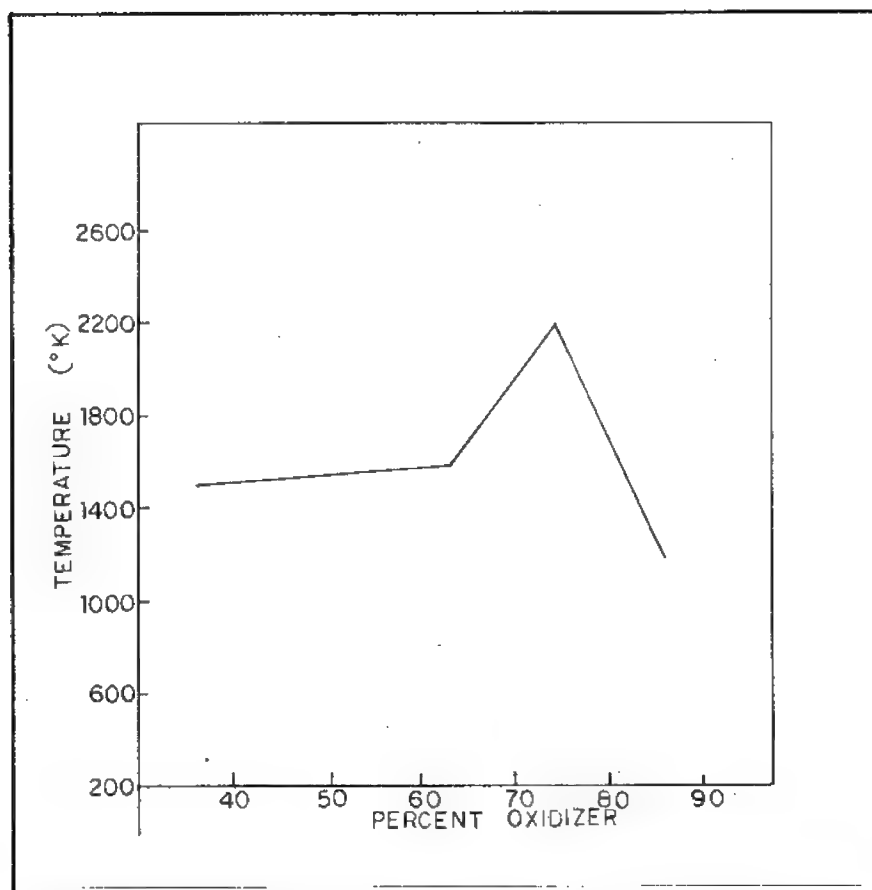
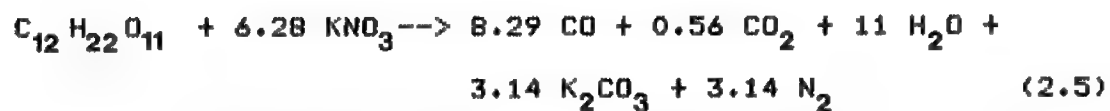


Figure 2.3. Adiabatic Flame Temperature as a function of O/F ratio.

The O/F ratio that is normally employed for this motor is 65/35. This gives the following combustion equation:



The performance analysis of the rocket motor is based upon this combustion ratio, unless stated otherwise.

Another parameter that is used extensively in ideal performance analysis is the average molecular weight of the gaseous

exhaust products, M' . This is calculated from the combustion equation and from the individual molecular weights of the products:

$$M' = \frac{n_i}{n_t} M'_i + \frac{n_j}{n_t} M'_j + \frac{n_k}{n_t} M'_k + \dots \quad (2.6)$$

where i , j and k are the individual constituents, n is the mole number, and where t refers to the total number of moles.

The ratio of specific heats, k , is another important quantity in the analysis of compressible fluid flow, which is the type of flow encountered in rocket nozzles, where k is defined by

$$k = \frac{C_p}{C_v} \quad (2.7)$$

where, for an ideal gas, k is a function of temperature only.

The value of k can be determined from knowledge of the specific heats, C_p , of the individual exhaust gases, where

$$k = \frac{C_p}{C_p - R'} \quad (2.8)$$

where R' is the universal gas constant, and

$$C_p = \frac{n_i}{n_t} C_{p_i} + \frac{n_j}{n_t} C_{p_j} + \frac{n_k}{n_t} C_{p_k} + \dots \quad (2.9)$$

However, complications arise in the flow through a supersonic nozzle where the temperature of the gases drop appreciably (as will be shown later), since C_p can be a strong function of temperature. It is possible to analyse such a flow utilizing a variable isentropic component [8], however, a sufficiently accurate

result can be obtained by calculating an average value of k for flow through the nozzle (see appendix B). Testins has found that for a 15° conical nozzle with a small area ratio (6 to 8), the variation in k does not have a pronounced effect [9]. This is similar to the nozzle type used for the motor under consideration.

The remaining combustion parameter to be determined is the burnrate, r (also called the surface regression rate). This is normally determined empirically, and is a function of the propellant composition and certain conditions within the combustion chamber. These conditions include propellant initial temperature, chamber pressure, and the velocity of the gaseous combustion products over the surface of the solid (erosive burning). It is necessary to combine a theoretical model together with empirical data in order to particularize the burnrate for a given propellant. The usual model is to approximate the burnrate as a function of pressure:

$$r = a P_0^n \quad (2.10)$$

where a and n are empirically determined constants, and P_0 is the combustion chamber pressure. The pressure exponent n , associated with the slope of the pressure-burnrate curve is almost independent of the propellant temperature. The coefficient a is a function of the initial propellant temperature, but not of pressure. From equation (2.10) it can be seen that the burnrate is very sensitive to the exponent n . High values of n give a rapid change of burnrate with pressure. This implies that even a small change in chamber pressure produces substantial changes in the quantity of

hot gases produced. As n approaches unity, burnrate and chamber pressure become very sensitive to one another and disastrous rises in chamber pressure can occur in a few milliseconds. When the value of n is low and becomes closer to zero, burning can become unstable and may even extinguish itself. Most production propellants have a pressure exponent ranging from 0.3 to 0.6 [10].

A correction is generally made for erosive burning where the increased burning can be accounted for by an empirical correction of the form:

$$r = r_0 (1 + ku) \quad (2.11)$$

where k is an empirical constant and u is the gas velocity. However, for this report this form of correction will not be applied. Instead, the burnrate is calculated with the erosive term taken directly into account.

2.2.3 Propellant Grain

The grain is prepared by casting the molten propellant into the desired shape. Several common configurations are shown in figure (2.4). The main criterion for choosing the geometry is to achieve the desired thrust-time characteristics. Thrust is a function of the instantaneous burning area which itself is dependant upon the initial grain configuration. The most common designs are those which achieve progressive, regressive, or neutral thrust-time curves, as shown in figure (2.5).

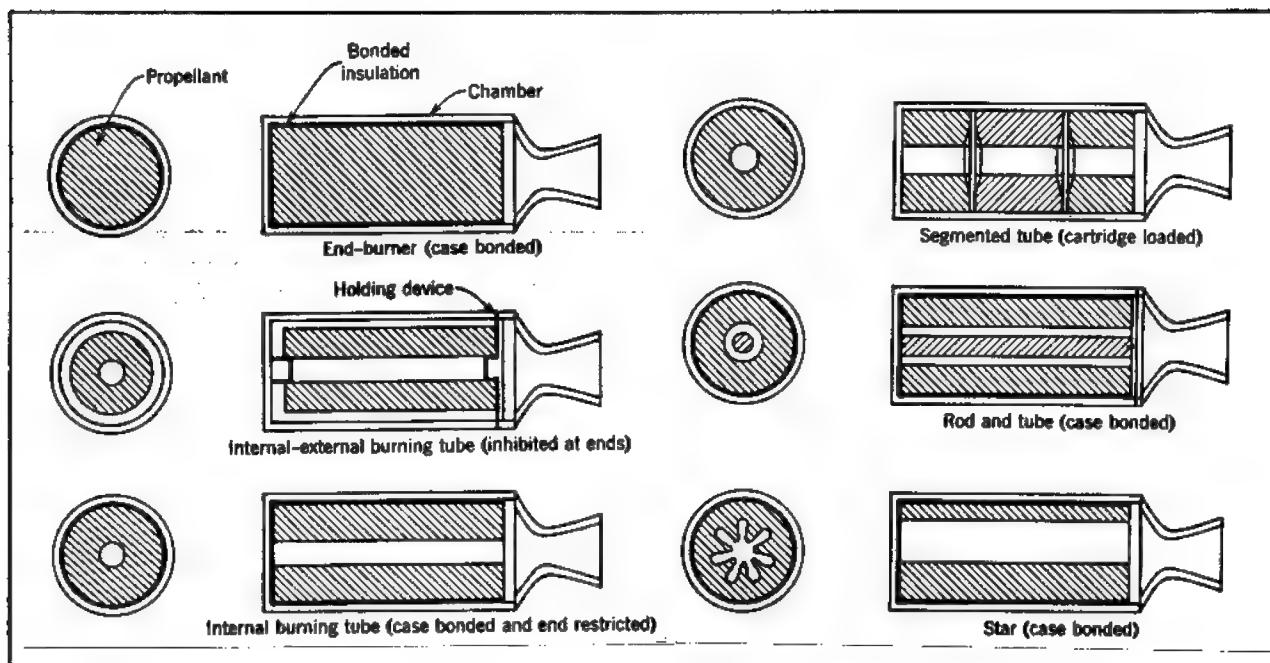


Figure 2.4. Several common grain configurations.

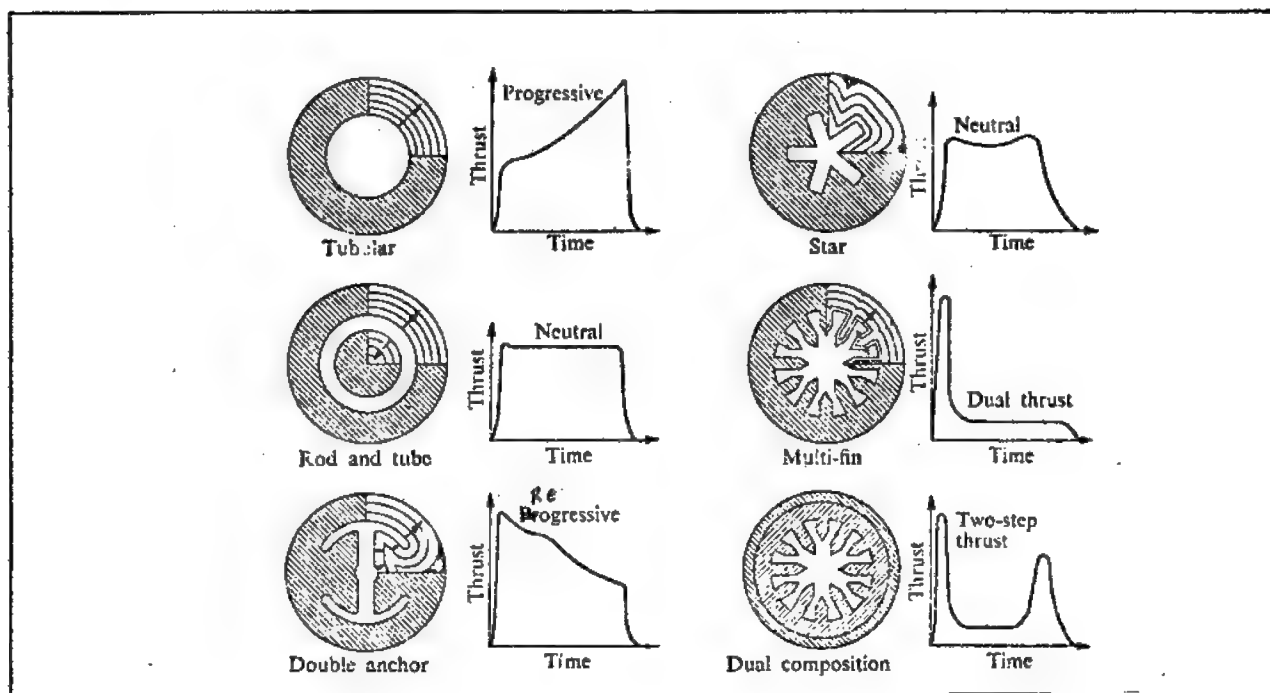


Figure 2.5. Internal burning grain designs with their thrust-time characteristics.

The grain configuration of the motor being considered is an internal-external burning tube, unrestricted at the ends. The theoretical thrust-time characteristic is slightly to moderately regressive, depending upon the length/diameter ratio.

The initial burning area, A_b , is given by:

$$A_b = \pi \left[L (D_o + D_i) + 0.5 (D_o - D_i^2) \right] \quad (2.12)$$

where L is the grain length, D_o and D_i are the grain outside and inside diameters, respectively.

A slightly modified version of this grain is also used in the motor (figure 2.6), where A_b is given by:

$$A_b = \pi \left\{ L_o (D_o + D_i) + \frac{L_c}{4} \left[3 D_o - \frac{1}{2} (D_i + D_o) \right] + \frac{1}{4} \left[\frac{1}{4} (D_o + D_i)^2 + D_o^2 - 2 D_i^2 \right] \right\} \quad (2.13)$$

These two expressions can be modified by utilizing equation (2.11) to obtain the instantaneous burning area, $A_b(t)$:

$$A_b(t) = \pi \left[(L - a t P_o^n) (D_o + D_i) + 0.5 (D_o - D_i^2) \right] \quad (2.14)$$

and

$$A_b(t) = \pi \left\{ (L_o - a t P_o^n) (D_o + D_i) + \frac{L_c}{4} \left[3 D_o - \frac{1}{2} (D_i + D_o) \right] + \frac{1}{4} \left[\frac{1}{4} (D_o + D_i)^2 + D_o^2 - 2 D_i^2 \right] \right\} \quad (2.15)$$

where t represents the time from initial burning. It is important to recognize that these expressions represent ideal burning which assumes that simultaneous ignition of all surfaces occurs at the beginning of the burn. As well the effects of increased burning surfaces due to flaws in the grain such as bubble holes and other flaws, as well as the effects of erosive burning are neglected.

These expressions are useful in calculating the chamber pressure and therefore thrust, as will be shown subsequently.

Propellant density, ρ_p and grain density, ρ_g are two additional properties that will prove useful. Ideally these two are identical but as a result of voids in the grain its density is somewhat lower.

The ideal propellant density is a function of the O/F ratio, since the density of the two constituents are different ($\rho_{\text{KNO}_3} = 2.11 \text{ g/cm}^3$ [11]; $\rho_{\text{sucrose}} = 1.58 \text{ g/cm}^3$ [12]). The ideal propellant density can be expressed as:

$$\frac{1}{\rho_p} = \frac{f_o}{\rho_o} + \frac{f_f}{\rho_f} = 1.888 \text{ g/cm}^3 \quad (2.16)$$

where f_o and f_f refer to the mass fraction of the oxidizer and fuel, respectively. For the 65/35 O/F ratio the actual propellant density has been found to be about 5 percent lower than ideal.

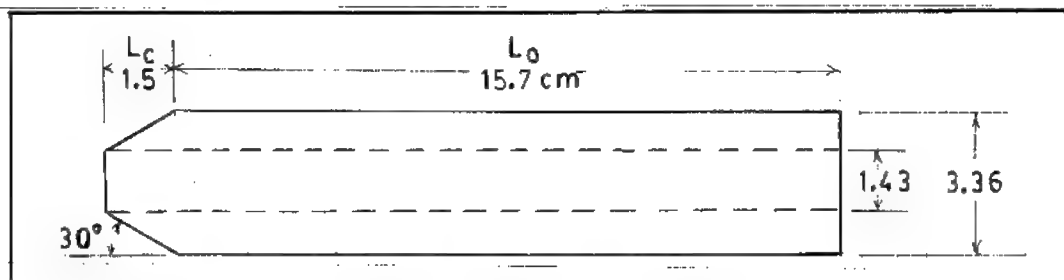


Figure 2.6. Modified Grain for the motor.

CHAPTER 3: NOZZLE THEORY

The analysis of a rocket nozzle flow involves the study of steady, one-dimensional compressible flow of an ideal gas. The actual flow differs somewhat from this simplified model particularly in regard to the presence of solid or liquid particles in the flow stream. The necessary modifications will be dealt with in the next chapter.

The analysis of compressible flow involves four equations of particular interest: continuity; momentum; energy; and the equation of state. These equations are applied to design a nozzle with the objective of accelerating the combustion gases (and particles) to as high an exit velocity as possible. This is achieved by designing the necessary nozzle profile with the condition that isentropic flow is to be aimed for. This necessitates minimizing frictional effects, flow disturbances, and conditions which can lead to shock losses. As well, heat losses would have to be minimized.

3.1 Nozzle Flow

In describing the state of a fluid at any point in a flow field it is convenient to employ the stagnation state as a reference state (the state characterized by a condition of zero velocity). Local isentropic stagnation properties are those properties that would be attained at any point in a flow field if the fluid at that point were decelerated from local conditions to zero velocity following a frictionless adiabatic, that is,

isentropic process.

The energy equation for an adiabatic flow between points x and y in which the decrease in enthalpy is equal to the increase in kinetic energy is given as:

$$h_x - h_y = \frac{1}{2} (v_y^2 - v_x^2) = C_p (T_x - T_y) \quad (3.1)$$

where h , v , and T are the enthalpy, velocity, and temperature, respectively.

The stagnation temperature T_o is defined from the energy equation as:

$$T_o = T + \frac{v^2}{2 C_p} \quad (3.2)$$

For an isentropic flow process, the following relations for stagnation conditions hold:

$$\frac{T_o}{T} = \left(\frac{p_o}{p} \right)^{\frac{k-1}{k}} = \left(\frac{\rho_o}{\rho} \right)^{k-1} \quad (3.3)$$

The local acoustic velocity of an ideal gas is defined as:

$$\bar{a} = \sqrt{k R T} \quad (3.4)$$

where R is the gas constant. The Mach number is defined as the ratio of flow velocity to the local acoustic velocity:

$$M = \frac{v}{\bar{a}} \quad (3.5)$$

From equations (3.2), (3.4), and (3.5) the total temperature Mach number relationship can be written:

$$\frac{T_0}{T} = 1 + \frac{k-1}{2} M^2 \quad (3.6)$$

It can be shown from the first and second laws of thermodynamics [13] that for an ideal gas undergoing an isentropic process assuming constant specific heats, that:

$$\frac{P}{\rho^{\frac{k}{k-1}}} = \text{constant} \quad (3.7)$$

From this result and from the equation of state, $P = \rho RT$, the stagnation pressure/density - Mach number relationship can be expressed as:

$$\frac{P_0}{P} = \left[1 + \frac{k-1}{2} M^2 \right]^{\frac{k}{k-1}} \quad (3.8)$$

$$\frac{\rho_0}{\rho} = \left[1 + \frac{k-1}{2} M^2 \right]^{\frac{1}{k-1}} \quad (3.9)$$

The use of equations (3.6), (3.8), and (3.9) allow each property (T, P, ρ) to be determined in a flow field if the Mach number and the stagnation properties are known. From the energy equation for an adiabatic flow (3.1) the stagnation enthalpy is defined:

$$h_0 = h + \frac{v^2}{2} \quad (3.10)$$

Physically, the stagnation enthalpy is the enthalpy that would be reached if the fluid were decelerated adiabatically to zero velocity. We note that the stagnation enthalpy is constant throughout an adiabatic flow field. Since the above stagnation properties (P_0 , ρ_0 and T_0) are related to the stagnation enthalpy by the specific heats and by the equation of state, it is apparent that each of these stagnation properties is constant throughout the adiabatic flowfield.

For steady, one-dimensional flow, the continuity equation can be written

$$\rho A v = \text{constant} = \rho^* A^* v^* \quad (3.11)$$

where A is the passage area, v is the velocity of the flow, and a starred (*) variable indicates critical conditions, or where M is unity.

From equations (3.4), (3.6), (3.9) and (3.11), it is possible to express the area ratio, A/A^* , in terms of the Mach number

$$\frac{A}{A^*} = \frac{1}{M} \left[\frac{1 + \frac{k-1}{2} M^2}{1 + \frac{k-1}{2}} \right]^{\frac{k+1}{2(k-1)}} \quad (3.12)$$

Figure (3.1) clearly shows that a converging-diverging passage with a section of minimum area is required to accelerate a flow from subsonic to supersonic speed. The critical point

where M becomes unity is seen to exist at the throat (point of minimum area) of the nozzle.

The variation of the properties during flow through the nozzle is illustrated in figure (3.2). From equation (3.1), the nozzle exit velocity, v_e , can be found by:

$$v_e = \sqrt{2(h_x - h_e) + \frac{v_x^2}{2}} \quad (3.13)$$

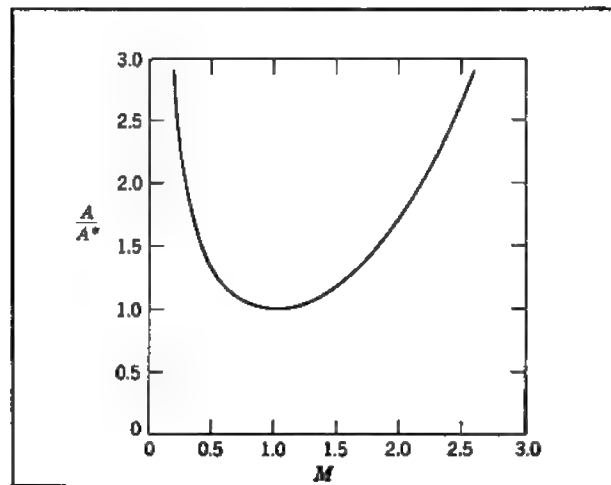


Figure 3.1. Variation of A/A^* with Mach number in isentropic flow for $k = 1.4$.

where h_x and v_x are the enthalpy and velocity at any point

within the nozzle. This relation applies to both ideal and nonideal rocket units. This equation can be rewritten with the aid of equations (2.8) and (3.3), (3.1)

$$v_e = \sqrt{\frac{2k}{k-1} \frac{R'T_0}{M'} \left[1 - \left(\frac{P_e}{P_0} \right)^{\frac{k-1}{k}} \right]} \quad (3.14)$$

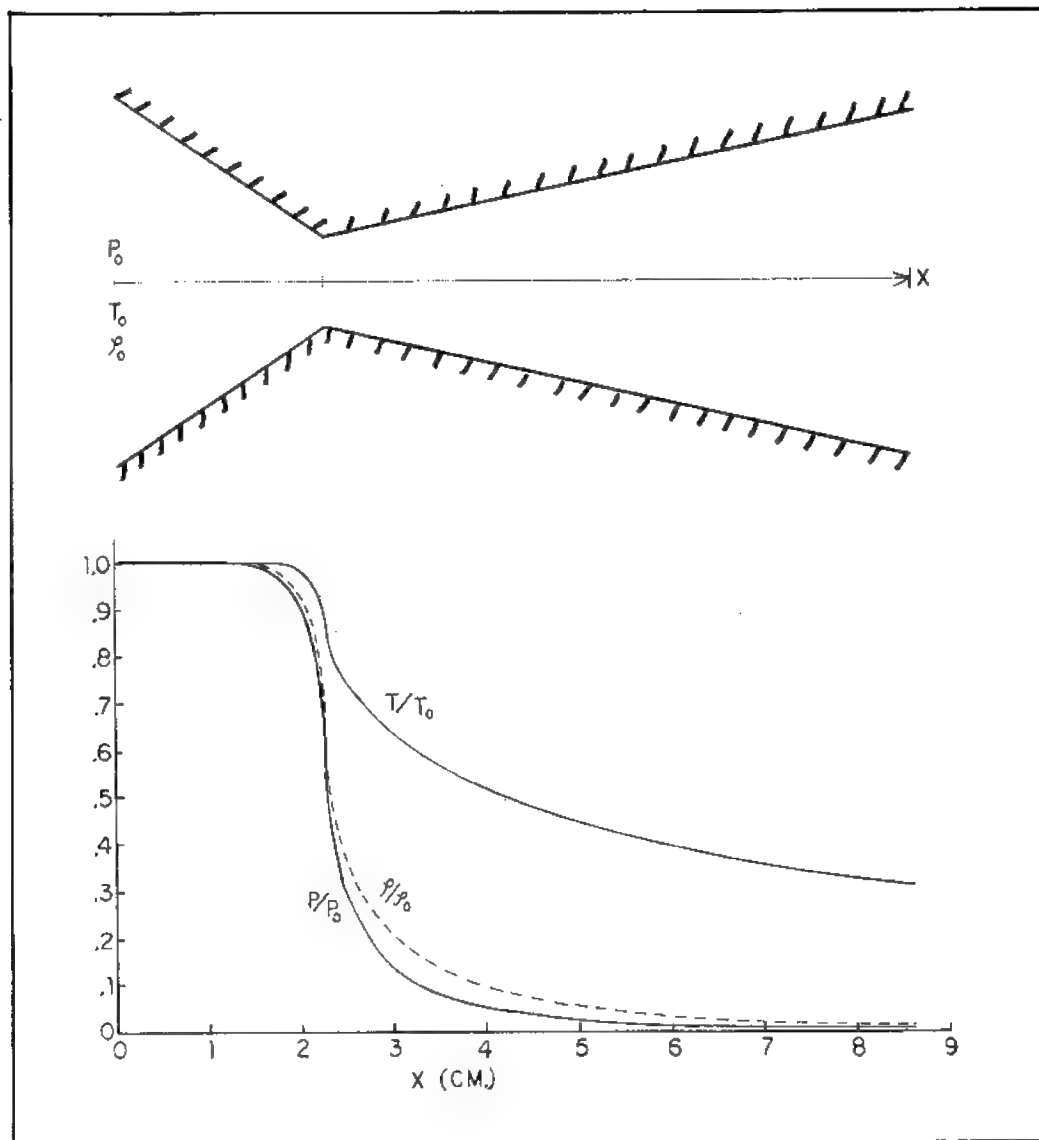


Fig. 3.2 . Variation of density ρ , pressure P , and temperature T , through the rocket nozzle.

for flow between the chamber, where stagnation conditions are assumed to exist, and the nozzle exit.

From this equation it is seen that the maximum exit velocity

occurs at an infinite pressure ratio P_0/P_e , or when exhausting into a vacuum.

The ratio between the throat and any downstream area at which pressure P_x prevails can be expressed as a function of the pressure ratio and k as follows, using equations (3.3), (3.9), (3.6), (3.4) and (3.14) (noting that at the throat M is unity):

$$\frac{A^*}{A_x} = \frac{\rho_x v_x}{\rho^* v^*} \quad (3.15a)$$

$$= \left(\frac{k+1}{2} \right)^{\frac{1}{k-1}} \left(\frac{P_x}{P_0} \right)^{\frac{1}{k}} \sqrt{\frac{k+1}{k-1} \left[1 - \left(\frac{P_x}{P_0} \right)^{\frac{k-1}{k}} \right]} \quad (3.15b)$$

This equation is important in that it allows the exit area, A_e , to be determined such that the exit pressure is equal to the ambient pressure P_a (typically 1 atm.). This is known as the design condition where it will later be shown that for such conditions maximum thrust is achieved. For this design, A_e/A^* is known as the optimum expansion ratio.

3.2 Rocket Performance Parameters

This section deals with the various performance parameters that are used to determine and compare the performance of solid propellant rocket motors. As well, modifications to the simplified model are considered to correct for real or actual performance. As well the effects of two phase flow are considered.

3.2.1 Thrust

The thrust F , of a rocket motor can be shown to be given by [14]:

$$F = \int P \, dA = \dot{m} v_e + (P_e - P_a) A_e \quad (3.16)$$

where the first expression represents the integral of the pressure forces acting on the chamber and nozzle projected on a plane normal to the nozzle axis (figure 3.3), $\dot{m}$ is the mass flowrate of the exhaust products and v_e is the exit velocity. The second term of the second expression is called the pressure thrust and is equal to zero for a nozzle with an optimum expansion ratio.

From continuity, equation (3.16) can be rewritten:

$$F = \rho^* v^* A^* v_e + (P_e - P_a) A_e \quad (3.17)$$

and modified using (3.14), (3.9), and (3.4) to yield:

$$F = A^* P_o \sqrt{\frac{2}{k-1} \left(\frac{2}{k+1} \right)^{\frac{k+1}{k-1}} \left[1 - \left(\frac{P_e}{P_o} \right)^{\frac{k-1}{k}} \right]} + (P_e - P_a) A_e \quad (3.18)$$

where the equation assumes k is constant throughout the expansion process. This equation shows that thrust is proportional to:

- i) throat area, A^*
- ii) chamber pressure, P_o
- iii) pressure ratio across the nozzle, P_o/P_e
- iv) specific heat ratio, k
- v) pressure thrust

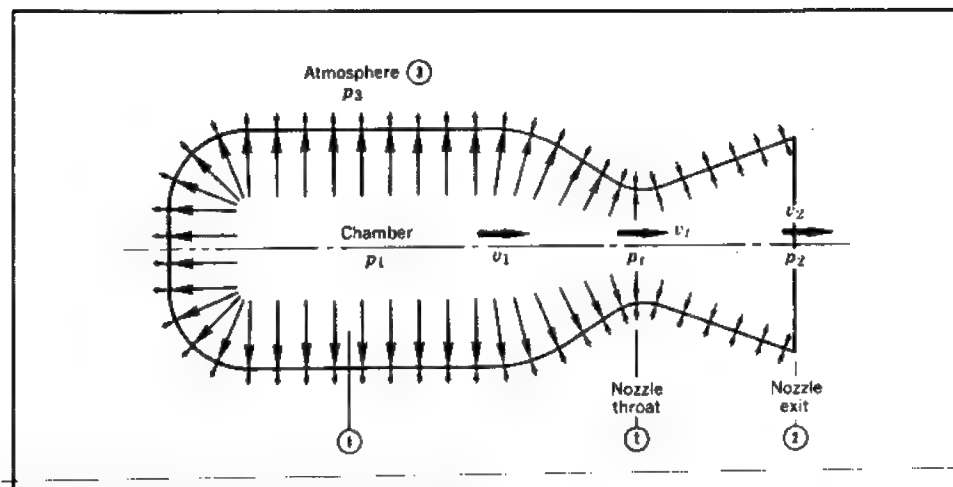


Fig. 3.3 . Pressure balance on chamber and nozzle walls; internal gas pressure is highest inside chamber and decreases steadily in nozzle, while external atmospheric pressure is uniform.

3.2.2 Thrust Coefficient

The thrust coefficient C_f , is defined as the thrust divided by the chamber pressure and throat area:

$$C_f = \frac{F}{P_o A^*} \quad (3.19)$$

The thrust coefficient determines the amplification of the thrust due to the gas expansion in the nozzle as compared to that would be exerted if the chamber pressure acted over the throat area only. From equation (3.18):

$$C_f = \sqrt{\frac{2k}{k-1} \left(\frac{2}{k+1} \right)^{\frac{k+1}{k-1}} \left[1 - \left(\frac{P_e}{P_o} \right)^{\frac{k-1}{k}} \right]} + \frac{(P_e - P_a)}{P_o} \frac{A_e}{A^*} \quad (3.20)$$

For any fixed pressure ratio P_e/P_o the maximum C_f can be found by taking the derivative $\frac{dC_f}{d(A_e/A^*)} = 0$. Therefore, the

maximum C_f will occur when $P_e = P_a$, or when the nozzle is designed

for optimum expansion (design condition).

Equation (3.19) is useful for comparing the measured C_f , to the theoretical value as determined from equation (3.20).

3.2.3 Characteristic Exhaust Velocity

The characteristic exhaust velocity, CV, is defined as $CV = c / C_f$, where c is the effective exhaust velocity:

$$c = \frac{F}{\dot{m}} = v_e + (P_e - P_a) \frac{A_e}{\dot{m}} \quad (3.21)$$

The CV can be expressed as a function of the gas properties in the combustion chamber using equations (3.21), (3.19), (3.1), (3.9) and (3.11):

$$CV = \sqrt{\frac{R T_o}{k \left(\frac{2}{k+1} \right)^{\frac{k+1}{k-1}}}} \quad (3.22)$$

CV is usually used as a figure of merit of a propellant combination and combustion chamber design, and is essentially independent of nozzle characteristics. This makes CV useful as a comparison for different propellants.

It is interesting to note that $CV \sim \sqrt{\frac{T_o}{M'}}$, recalling that $R = R' / M'$. A high value of CV is desirable; however a high value of T_o might not be. High combustion temperatures may cause excessive heating of the parts of the rocket motor that are exposed directly to hot gases. Nozzle erosion can be severe with conven-

tional materials particularly when burn durations exceed several seconds. For this reason it may be more desirable to increase CV by reducing M' rather than increasing T_0 . This can often be accomplished by using a propellant that is on the fuel rich side of stoichiometric.

The CV can also be considered an expression of the impetus ($n R T$) of the propellant. This is useful for determining a propellant CV in the laboratory by means of a closed bomb (constant volume) measurement [15].

3.2.4 Impulse

The impulse (or total impulse), I , is the integral of the thrust over the operating duration, t :

$$I = \int_0^{t_b} F dt \quad (3.23)$$

or the area under the thrust-time curve (figure 3.4).

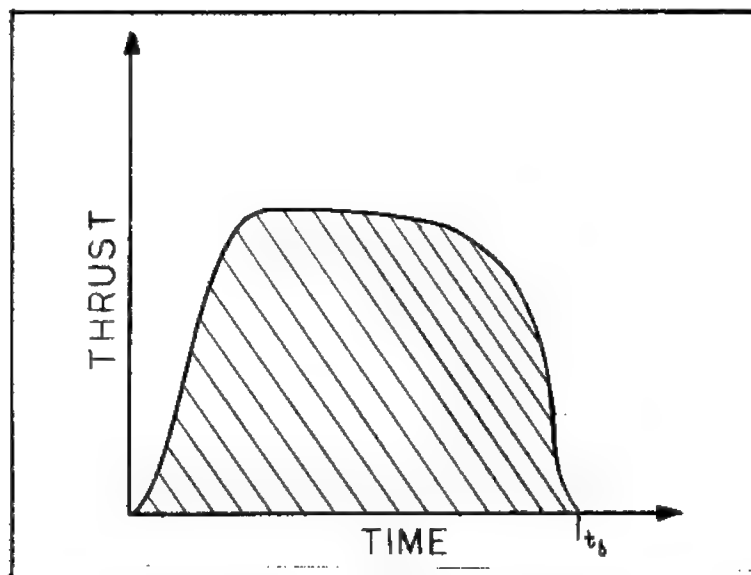


Figure 3.4. Area under thrust-time curve represents the total impulse of the motor.

The operating duration is approximately equal to the burn time, t_b , for motors with longer burn times ($t_b > \text{several seconds}$). For short burntimes such as those associated with small motors the duration of thrust would be considered the burntime plus the time duration for the residual gases to exhaust from the combustion chamber after burnout. The total impulse would then be given by [16]:

$$I = \frac{2}{k+1} C_f P_o A^* \tau \quad (3.24)$$

where τ is a time constant given by

$$\frac{1}{\tau} = \frac{A^* P_o}{\rho_o V_o CV}$$

where n is the burnrate exponent and V_o is the chamber volume.

The specific impulse is one of the most important performance parameters used in rocket research. It is defined as the thrust that can be obtained from an equivalent rocket which has a propellant weight flow of unity. It is given by:

$$I_{sp} = \frac{F}{\dot{W}} = \frac{c}{g} \quad (3.25)$$

With solid rocket motors it is difficult to measure the weight flowrate, so the average specific impulse is usually employed, where $\bar{I}_{sp} = I / t$.

The ideal specific impulse can be calculated for a given propellant and motor combination from equations (3.25) and (3.14):

$$I_{SP} = \sqrt{\frac{2k}{g^2(k-1)} \frac{R'}{M'} T_o \left[1 - \left(\frac{P_e}{P_o} \right)^{\frac{k-1}{k}} \right]} \quad (3.26)$$

For comparison purposes P_o is usually taken as 1000 psia and P_e as 14.7 psia, by convention. A simplified semi-empirical method for determining specific impulse was developed by Free and Sawyer [17] with a claimed accuracy of 3 to 5 percent.

Recalling that CV is also a measure of the propellant energy, the actual specific impulse can be determined by utilizing the ballistic bomb method as previously mentioned, where

I_{SP} is related to CV by:

$$I_{SP} = \frac{CV C_f}{g} \quad (3.27)$$

however, requiring the use of the ideal C_f unless the actual value is known.

3.2.5 Chamber Pressure

The combustion chamber of a solid propellant rocket is essentially a high pressure tank containing the entire solid mass of propellant. Combustion proceeds from the surface of the grain where the rate of gas generation is equal to the rate of consumption of solid material, in the ideal case where only gaseous products result (no solids or liquids):

$$\dot{m}_g = \rho_p A_b r \quad (3.28)$$

where $\dot{m}_g$ is the gas generation rate, A_b is the area of the burning surface, and r is the surface regression rate (burnrate). The rate at which gas is stored within the combustion chamber is given by:

$$\frac{dm_s}{dt} = \frac{d}{dt}(\rho_o v_o) \quad (3.29)$$

where ρ_o is the instantaneous gas density and v_o is the instantaneous chamber volume. The use of these two equations as well as the expression for the rate at which gas flows through the nozzle and the expression for burnrate, leads to the following expression for chamber pressure [18]:

$$P_o = \left[\frac{A_b}{A^*} \frac{a(\rho_p - \rho_g)}{\sqrt{\frac{k}{RT_o} \left(\frac{2}{k+1} \right)^{\frac{k+1}{k-1}}}} \right]^{\frac{1}{1-n}} \quad (3.30)$$

For constant P_o and therefore constant thrust it is clear that the burning area must remain constant, such as with a neutral burning grain. The instantaneous chamber pressure can be given by the same expression, in which A_b is the instantaneous burning area. The variation of A_b with time depends upon the burning rate and the initial geometry of the propellant grain.

For short burn time motors such as the one under consideration (less than one second burntime) the duration under which steady-state conditions exist, where equation (3.30) applies, accounts for perhaps one-third to one-half of the total thrust time. Prior to ignition, the combustion chamber is filled with cool air of ambient pressure. The start-up period where the pressure builds up to the steady-state value is a complex problem that has been studied by Von Karman and Malina [19]. They have developed an expression for the pressure as a function

of time during start-up, given by:

$$P = P_0 (1 - e^{-t/\tau}) \quad (3.31)$$

where P is the instantaneous pressure at time t and where τ is previously defined.

Immediately after burnout the combustion chamber remains filled with high pressure gas. The expression for chamber pressure is given as a function of time by [20]:

$$P = P_0 \left(1 + \frac{k-1}{2} \frac{t}{\tau} \right)^{\frac{-2k}{k-1}} \quad (3.32)$$

3.2.6 Corrections for Two Phase Flow

The previous analysis of nozzle flow and performance parameters considered the ideal case where the working fluid is a pure gas. This analysis is fine for the case where there is no solid or liquid particles in the exhaust (such as for liquid propellants) but must be modified for the case where such particles are present. The presence of particles is detrimental to the overall rocket performance, with the extent dependant upon several factors.

One of these factors is motor size. It has been found by Gilbert, Allport, and Dunlop [21] that for large motors ($>10^4$ pounds thrust) the overall specific impulse losses are low, but for small motors ($\approx 10^2$ pounds thrust) the losses can be significant. The motor under consideration is of this magnitude thrust making corrections for two phase flow necessary.

The losses in performance are due to two factors:

i) velocity lag, where the particles are not accelerated through the nozzle to the same velocity as the gases; and

ii) thermal lag, where the particles are not in thermal equilibrium with the surrounding gases.

It has been found that the latter has a small effect, usually allowing the assumption that the particles are in thermal equilibrium [22].

The starting point in the consideration of the effects of solid (liquid) matter formation is with the equation relating to the gas generation rate (equation 3.28). This equation becomes:

$$\dot{m}_g = (1 - X) \rho_p A_b r \quad (3.33)$$

where X is the mass fraction of solid (liquid) in the exhaust.

As a consequence, this would change the equation for steady-state chamber pressure to:

$$P_o = \left[\frac{A_b}{A^*} \frac{(1 - X) \rho_p r}{\sqrt{\frac{k}{RT_o} \left(\frac{2}{k+1} \right)^{\frac{k+1}{k-1}}}} \right]^{\frac{1}{1-n}} \quad (3.34)$$

where the chamber pressure is reduced by a factor of $(1 - X)$.

The expression for specific impulse would change to account for the difference between particle and gas velocity:

$$I_{SP} = \frac{1}{g} \frac{\sum \dot{m}_i v_i}{\sum \dot{m}_i} \quad (3.35)$$

where i denotes a particular constituent. If all the particles

are assumed to have the same velocity this can be expressed as:

$$I_{SP} = \frac{1}{g} \frac{(\dot{m}_s v_s + \dot{m}_g v_g)}{\dot{m}_s + \dot{m}_g} \quad (3.36)$$

To accurately describe the effects of particles on a rocket's performance requires knowledge of heat transfer properties and drag processes of the particles in order to express v_s and T_s in terms of the overall flow properties. It is possible, however, to derive the limiting cases to determine the least and most detrimental effects.

In the first model, the particles are assumed to be very small, where $T_s \approx T_g$ and $v_s \approx v_g$. This would give the following expression for exhaust velocity (see appendix c for derivation):

$$v_e = \sqrt{2 T_o \left[C_s X + (1 - X) \frac{k R'}{(k - 1) M'} \right] \left[1 - \left(\frac{P_e}{P_o} \right)^m \right]} \quad (3.37)$$

$$\text{where} \quad m = \left[\frac{X C_s M'}{(1 - X) R'} + \frac{k}{k - 1} \right]^{-1}$$

where C_s is the average specific heat of the solid (liquid) matter. Note that this equation reduces to the familiar form (3.14) when $X = 0$ (no particles).

The expression for thrust would be modified to this form (see appendix D for derivation):

$$F_i = A P_o \frac{1}{1 - X} \sqrt{\frac{2 k M'}{R'} \left(\frac{2}{k + 1} \right)^{\frac{k+1}{k-1}} \left[X C_s + (1 - X) \frac{k R'}{(k - 1) M'} \right] \left[1 - \left(\frac{P_e}{P_o} \right)^m \right]} + (P_e - P_o) A_e \quad (3.38)$$

Note again that the equation reduces to the familiar form (3.18) when $X = 0$.

The expression for the thrust coefficient is derived in the same manner, yielding the expression:

$$C_f = \frac{1}{1-X} \sqrt{2k \left(\frac{2}{k+1} \right)^{\frac{k+1}{k-1}} \left[\frac{M' X C_s}{R'} + \frac{k(1-X)}{k-1} \right] \left[1 - \left(\frac{P_o}{P_o'} \right)^m \right]} \quad (3.39)$$

It is interesting to note that the thrust coefficient increases with the presence of particles compared to the gas only value. This equation implies a 75 percent increase in the thrust coefficient for a typical propellant where $X = 0.45$.

The second limiting case models the particles such that the particle velocity and temperature remain essentially constant. This implies a combination of relatively large physical size, slow heat transfer, and low drag. With such a model, the expressions for the performance parameters remain identical to the original (gas only) expressions, i.e. the gas flow is not affected by the presence of particles. However, the specific impulse is reduced. From equation (3.36) where $m_s v_s \ll m_g v_g$:

$$I_{sp} = \frac{1}{g} (1-X) v_g \quad (3.40)$$

Therefore, the specific impulse is reduced by a factor of $1-X$.

Research conducted into two phase flow also indicates that the nozzle type has a bearing on the effect of particles in the exhaust stream [23]. As well, it has been found that particle

size (and distribution) is independent of motor size, and is therefore largely a property of the particular propellant [24]. Another detrimental effect of liquid particles is the tendency for the material to build up on the nozzle surfaces, reducing the effective nozzle area. Additional losses are introduced by the resulting rough surface which increases skin friction [25].

For the motor being considered, the losses due to particle presence will be accounted for in the energy conversion factor, discussed below.

3.2.7 Corrections for Real Nozzles

The preceding analysis considers ideal rockets, which of course do not exist. The ideal case represents the maximum performance that can be attained, with the actual performance being reduced by a number of factors. These factors are accounted for in real nozzles by using various correction factors.

Conical nozzles require a factor to correct for the non-axial component of the exit gas velocity as a result of the divergence angle, defined as 2α (where α is the half angle). This factor λ , is given by:

$$\lambda = \frac{1}{2} (1 + \cos \alpha) \quad (3.41)$$

As well, the flow in an actual nozzle differs from ideal because of frictional effects, heat transfer, imperfect gases, nonaxial flow, nonuniformity of the working fluid and flow distribution, and the effects of particles. The degree of de-

parture is indicated by the energy conversion efficiency of the nozzle. This is defined as the ratio of the kinetic energy per unit of flow of the Jet leaving the nozzle, to the kinetic energy per unit flow of a hypothetical ideal Jet leaving an ideal nozzle that is supplied with the same working fluid at the same initial state and velocity and expands to the same exit pressure as the real nozzle. This is expressed as:

$$e = \frac{\frac{1}{2} v_{0a}^2}{\frac{1}{2} v_{0i}^2} \quad (3.42)$$

where the subscripts i and a refer to the ideal and actual states and e denotes the energy conversion efficiency.

The velocity correction factor, ζ_v , is defined as the square root of e. For most production motors, the value lies between 0.85 and 0.98. This factor is also approximately the ratio of the actual to ideal specific impulse.

The discharge correction factor ζ_d , is defined as the ratio of the mass flow rate in a real nozzle to that of an ideal nozzle that expands an identical working fluid from the same initial conditions to the same exit pressure:

$$\zeta_d = \frac{\dot{m}_a}{\dot{m}_i} = \frac{\dot{m}_a c}{F} \quad (3.43)$$

The value of the discharge factor is often larger than unity, because the actual flow can be larger than theoretical for the following reasons:

- i) the molecular weight of the gases increases slightly

when flowing through a nozzle, thereby changing their density.

ii) some heat is transferred to the nozzle walls, lowering the gas temperature, increasing its density.

iii) the specific heat and other gas properties change in an actual nozzle in such a way as to slightly increase the value of the discharge coefficient.

iv) incomplete combustion increases the density of the exhaust gases.

These correction factors result in a thrust lower than for the ideal case:

$$F_a = \zeta_v \zeta_d \lambda F_i \quad (3.44)$$

$$F_a = \zeta_v \zeta_d \lambda C_{f_i} P_{o_e}^* A \quad (3.45)$$

CHAPTER 4: EXPERIMENTAL TECHNIQUE

4.1 Motor

The motor under consideration is illustrated in the figure below (figure 4.1), with a complete description given in appendix (E).

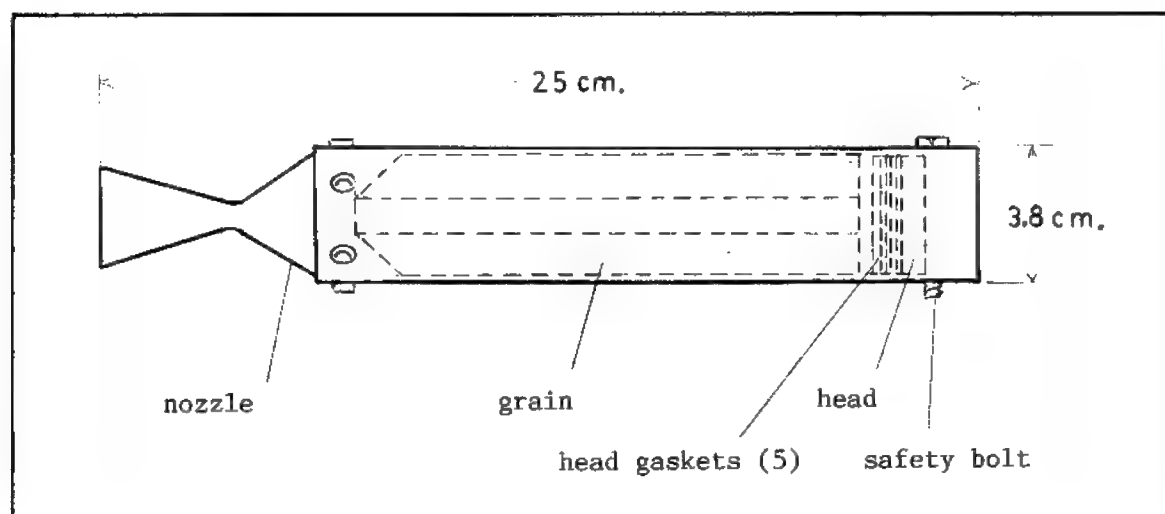


Figure 4.1. Actual motor used in testing.

The standard nozzle has a conical profile with a 12 degree divergence half angle. It was fabricated from mild steel bar stock turned and bored to the desired dimensions on a standard lathe. The nozzle contour is rounded at the throat to avoid sharp discontinuities which could lead to shock losses. As well, the entire nozzle inside surface is polished to reduce friction losses.

The nozzle is retained to the combustion chamber by six high strength allen screws.

The head of the motor is readily removable to allow for easy loading of the grain. Constructed as well from mild steel, the head is retained by a single safety bolt ($3/16 \times 1\ 1/2$ in.; grade 8) designed to shear if the chamber pressure should attain an unsafe level (17 MPa.). The combustion gases are sealed from escaping around the head by five asbestos-fibre composite gaskets. The nozzle is sealed against the combustion chamber by utilizing a tool which rolls a circular die under high pressure around the perimeter of the chamber outside wall against the nozzle section forward of the retention groove. This effectively seals the chamber wall against the nozzle to reduce gas leakage.

4.2 Cubic Nozzle Profile

Although the exact nozzle profile is not critical for good performance as the flow occurs in a region of favourable pressure gradient, optimum performance is achieved by designing the nozzle by the method of characteristics. A thorough discussion of this technique is presented in references [26] and [27]. This method is lengthy and complex and will not be considered here. However, an approximation to this design is that of a cubic profile where the nozzle contour follows the curve of a cubic equation [28].

A nozzle was constructed with the aid of a computer numerically controlled lathe. The criterion for design was based on the same throat, entrance and exit areas as the conical nozzle. The convergent section was based on a maximum half angle of 30 degrees.

and a maximum divergent angle of 20 degrees. Figure (4.2) shows the dimensions of the nozzle where the equations of the contours are:

$$R = 1.748 - 0.762 x^2 + 0.226 x^3 \text{ cm. (Region I)} \quad (4.1)$$

$$R = 0.461 + 0.201 x^2 - 0.037 x^3 \text{ cm. (Region II)} \quad (4.2)$$

where R is the inside radius.

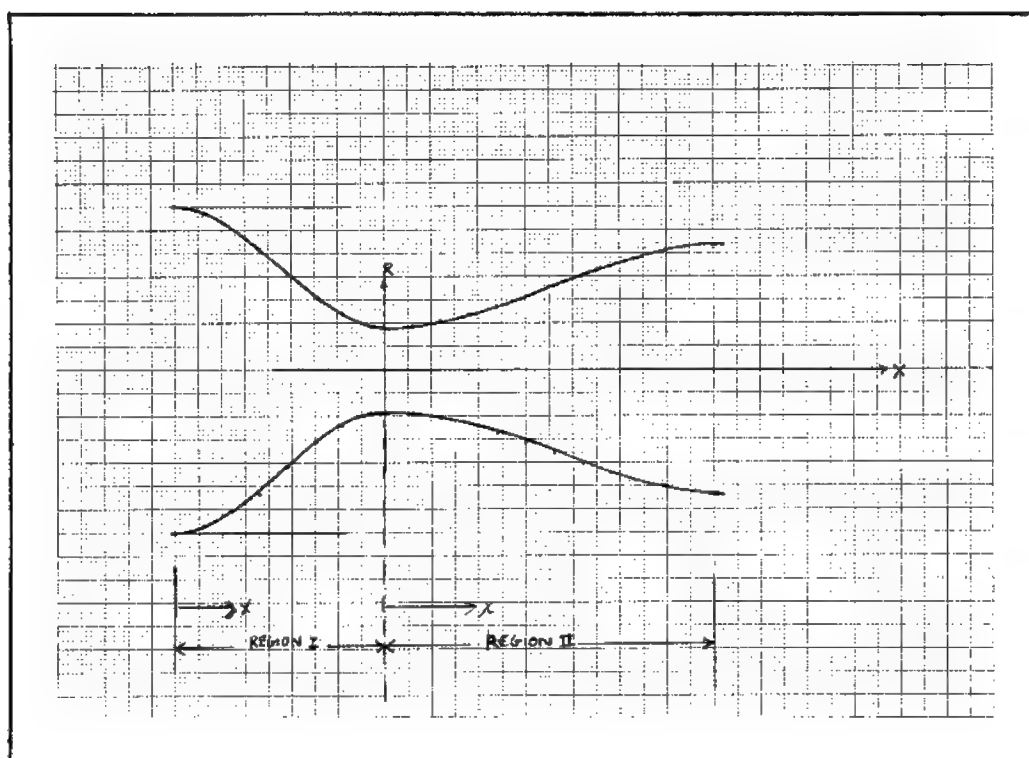


Figure 4.2. Cubic nozzle profile.

This nozzle was constructed of cold rolled mild steel bar stock, and was highly polished on the flow surfaces. The divergence exit angle was machined to zero degrees, to eliminate radial flow losses (non-truncated).

It should be noted that while the critical dimensions of the cubic nozzle are coincident with the conical nozzle, the overall length is about 17 percent shorter.

4.3 Propellant Preparation

As stated previously, the standard propellant combination is 65% oxidizer (KNO_3) and 35% fuel (sucrose). The propellant (grain) preparation technique for any O/F ratio is identical, however.

In order to ensure complete mixing and intimate contact between the oxidizer and fuel crystals, each constituent must be ground to a small particle size. Commercial oxidizer crystals are ground according to the following standard ranges:

Coarse	400 to 600 μ ($1 \mu = 10^{-6} \text{ m}$)
Medium	50 to 200 μ
Fine	5 to 15 μ
Ultrafine	submicron to 5 μ

The commercially obtained potassium nitrate oxidizer has an average particle size of 250 μ . The oxidizer is subsequently ground down to an average particle size of 100 μ . The sucrose is obtained in powder form with typical particle size of approximately 10 μ .

It has been found that the particle size of the oxidizer has an important influence on both the burning rate and more importantly on the impulse delivered. Therefore, careful attempt was made in the experimentation to obtain consistency in oxidizer particle size.

After accurately weighing the desired quantities of both constituents, they are placed in an electric rotating drum mixer where mixing occurs for twenty hours.

The grain casting procedure involves heating the mixture in a container maintained at a temperature of 190 to 200 degrees celcius by immersion in an electrically heated oil bath. The sucrose, (m.p. 182 degrees C.) combines with the non-melting KNO_3 (m.p. 441 C.), to form a slurry. Decomposition of the sucrose initiates as well at this temperature being both a function of of temperature and time. Therefore, the temperature has to be maintained within this range, and for the minimum time duration necessary for the entire mixture to melt. This decomposition (caramelization), if kept in check, has been found to have no detrimental effect on the propellant performance. Investigation of the effects of low degree caramelization of sucrose indicates that there is no significant mass change, which

is important since the occurrence of such would alter the effective O/F ratio. As well, it has been found that less than 1 percent moisture is absorbed by the finely divided sugar, which is again an important consideration for any moisture initially present would be given up upon heating.

Once the slurry has become sufficiently fluid, it is cast into the desired shape. This shape (hollow cylindrical) is achieved by pouring the slurry into a lubricated cylindrical mould, of slightly smaller diameter than the combustion chamber. A lubricated bore rod is inserted down the central axis. After allowing to cool and harden for approximately forty-five minutes, the grain is removed, trimmed to size, weighed and measured. The grain is then stored in a sealed container. This is to prevent exposure to the open air for the propellant in its final form is very hygroscopic.

4.4 Varying O/F Ratio

A number of grains were prepared to determine the effect of varying O/F ratios upon the motors impulse. As well, several small cylinders (1.5 x 10 cm) of varying O/F ratios were prepared for burnrate testing.

4.5 Burnrate testing

In order to quantify burnrates under conditions of one atmosphere and at room temperature, several samples of different O/F ratios were prepared. The samples were cast into cylinders

as indicated earlier. These cylinders were mounted upright by adhering one end to a base. A specified length was marked off, typically 7 cm. The top end was then ignited by a flat surfaced tool, heated prior in a flame. As the sample burned down, the elapsed time was measured between marks. This type of burnrate testing gives results for conditions of constant temperature propellant and non-erosive burning.

More complete burnrate testing would involve determining the dependance of burnrate on pressure. In order to conduct such testing, it would be necessary to construct a suitable vessel that would allow burning to occur at constant pressure. Such an investigation has not yet been conducted, however. Research of this type has indicated that the burnrate measured by this technique is not in complete agreement with the actual burnrate that occurs in the actual motor, being about 7 percent higher [29]. No explanation could be found for this discrepancy.

4.6 Motor Static Testing

Static testing of the motor (figure 4.3) to obtain the thrust-time characteristics is conducted on a specially built test rig (figure 4.4). A remotely located Heath H-8 microcomputer is used for data acquisition, with the measured signal sent to it via a 50 metre shielded cable. The acquisition system is potentially capable of handling up to four channels of data input (e.g. thrust, chamber pressure, temperature, and wall stress). For this series of tests, only thrust measurements were attempted, however.

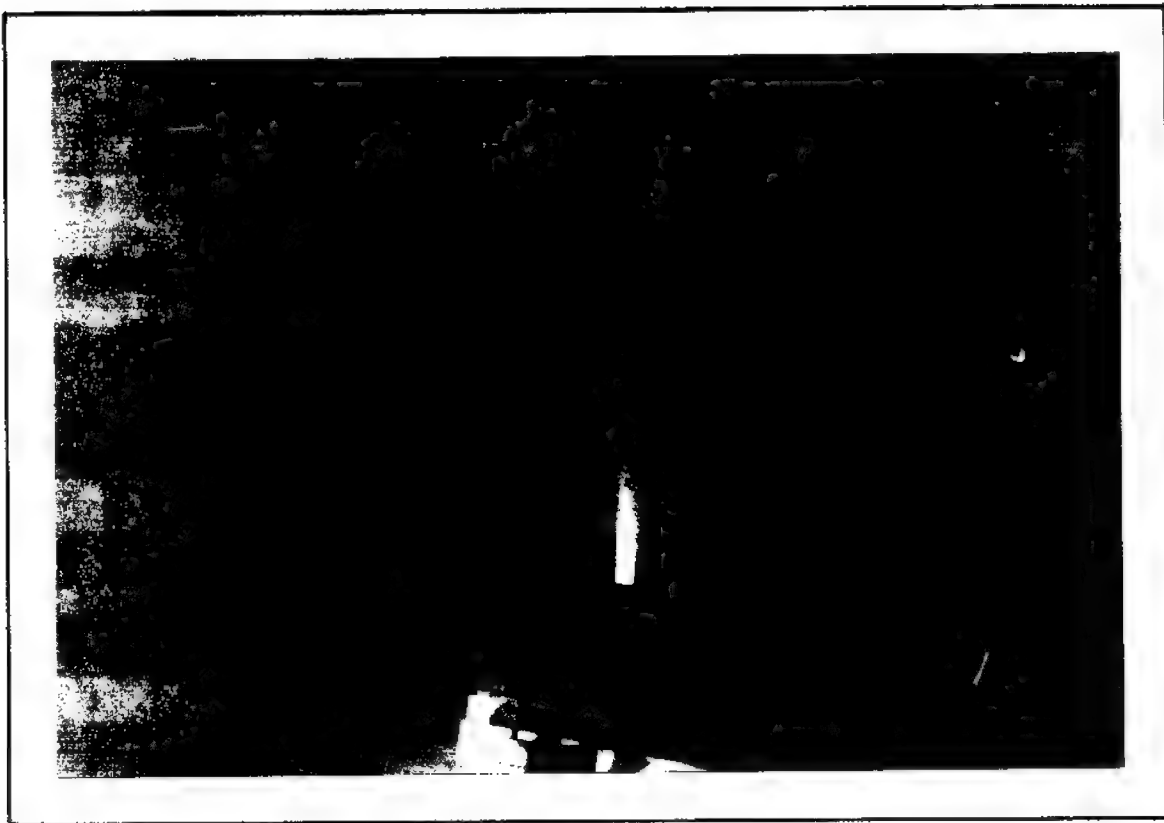
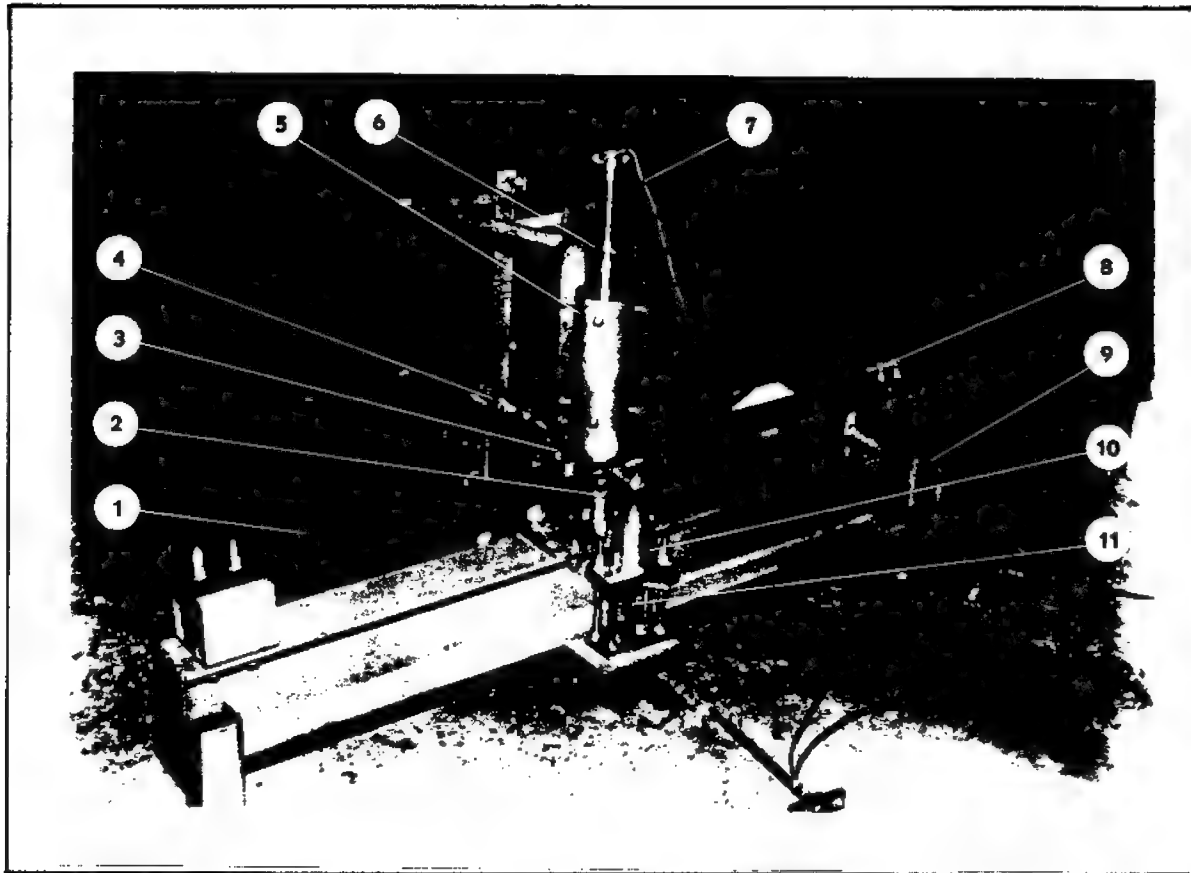


Figure 4.3. Static firing of the rocket motor mounted in test rig.

4.6.1 Test Rig Construction

The frame of the rig is constructed of heavy steel I-beam to eliminate undesired deflections under high thrust. Although the maximum thrusts endured by the motor under consideration were 1.1 kN (250 lbs.), the rig can handle much larger values of thrust.

The motor is mounted vertically, nozzle upward, in a holder designed to allow for the small vertical motion encountered by the motor during firing.



- | | |
|----------------------|-----------------------------|
| 1. Deflection bar | 7. Electrical igniter |
| 2. Lower stop | 8. Deflection bar tensioner |
| 3. Upper stop | 9. Frame |
| 4. Strain transducer | 10. Hydraulic damper |
| 5. Motor mount | 11. Damper adjust valve |
| 6. Rocket motor | |

Figure 4.4. Rocket motor static testing rig.

The frame supports a double cantilever deflection bar assembly. It is against this bar that the motor acts during firing resulting in the bar deflecting. The thrust is therefore transduced into a lineal deflection. In order to obtain high resolution of the thrust transduction, it is desirable to have a relatively large deflection of the bar. This maximum deflection is determined by the transducer unit, having been determined to be 0.635 cm. (1/4 in.). By knowing the maximum expected thrust of motor it is possible to choose the bar-cantilever assembly to achieve this (for any motor).

The deflection y for a double cantilever assembly is given by [30]:

$$y = \frac{F L^3}{192 E I} \quad (4.3)$$

where F is the force (thrust), E is the Young's modulus of the bar material, I is the moment of inertia of the bar, and L is the bar length. For a rectangular bar of thickness d and width b , this can be rewritten as:

$$y = \frac{F L^3}{16 E b d^3} \quad (4.4)$$

The deflection is therefore proportional to L/D cubed and inversely proportional to the width. Therefore, the choice of the bar gives great flexibility in achieving the maximum deflection, $y_{\max}$.

A lower stop is placed directly below the centre of the

bar to prevent damage to the transducer unit in case of over-thrusting, or a blowout of the motor head (see figure 4.5).

The thrust induced displacement of the transducer bar results in an equal displacement of the end of the transducer unit, located directly under the motor mount. This transducer unit uses a strain gauge bridge fastened to its fixed end. This strain gauge bridge forms the fundamental component of the data acquisition system, illustrated in figure (4.6). The amplifier

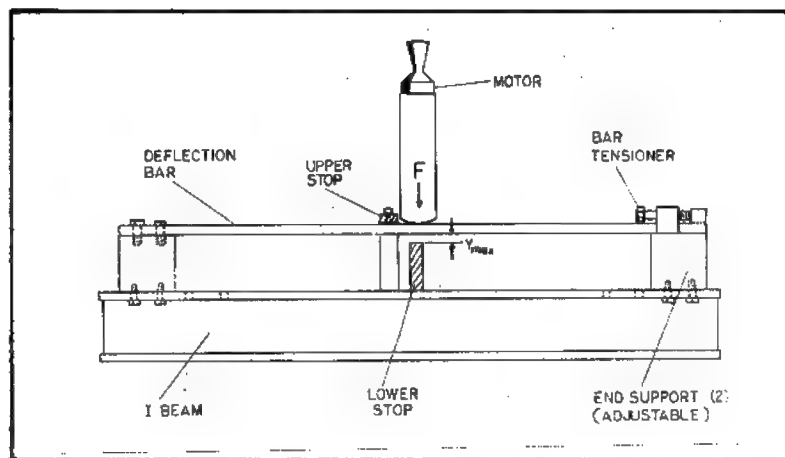


Figure 4.5. Deflection bar assembly of static test rig.

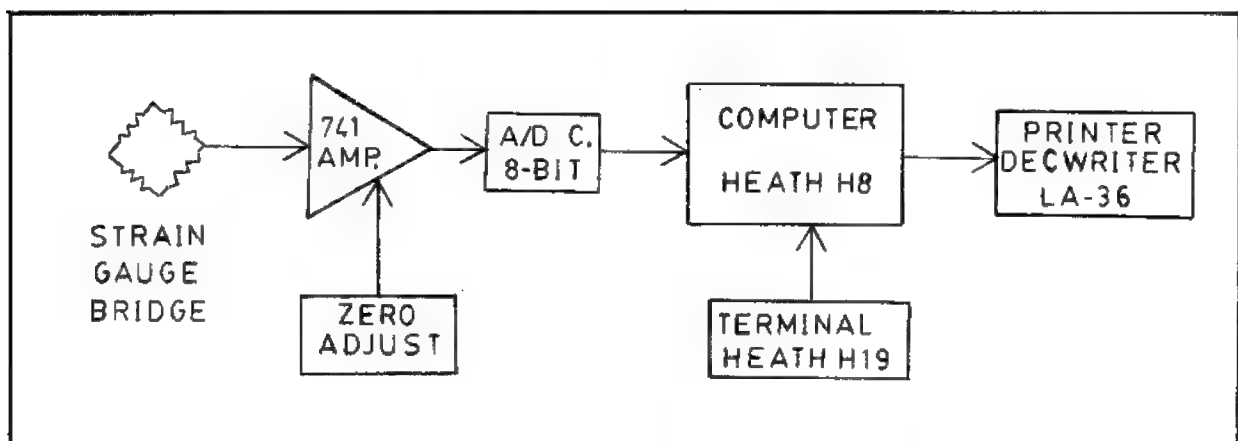


Figure 4.6. Data acquisition system used to measure motor thrust-time characteristics.

circuit is reproduced in Appendix (F). The A/D converter is an 8-bit, four channel unit capable of sampling 581 points per second (samples every 1.72 milliseconds). These points are stored in the computer memory for post-processing.

Due to the spring-mass nature of the deflection bar assembly, it was found necessary to provide adequate damping to reduce oscillations of the deflection bar. A variable hydraulic damper was designed and built to allow a large degree of flexibility of damping. The damper is detailed in figure (4.7). The operation of

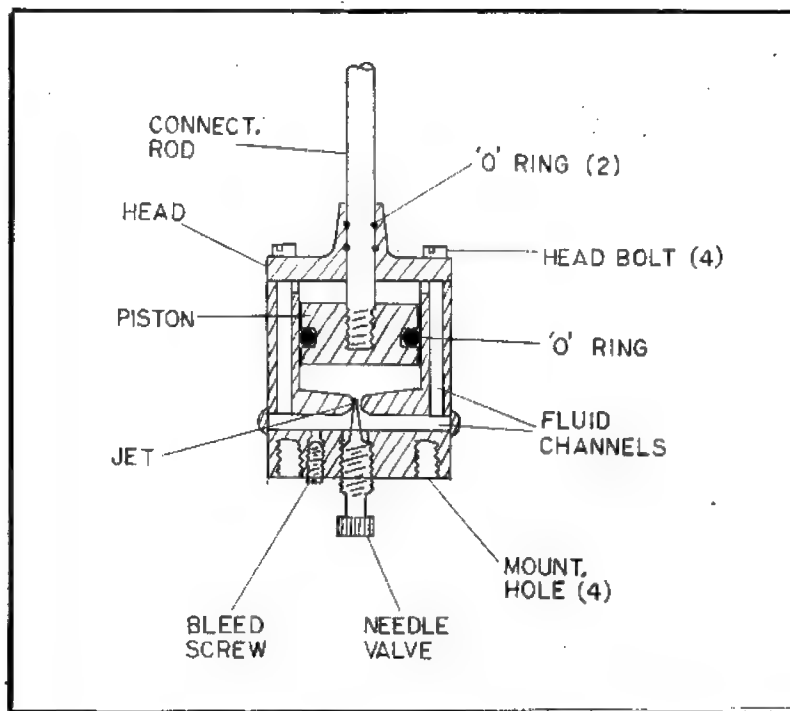


Figure 4.7. Hydraulic damper used with test rig.

the device is straightforward. A connecting rod with a piston at one end is attached to the deflection bar. The reversed vertical

motion of the rod alternately forces fluid from and back into the fluid chamber through a small jet. The effective area of the jet is controlled by an adjustable needle valve, the adjustment of which governs the degree of damping. This allows for a wide range of damping from essentially nil (needle turned out) to complete damping (needle turned in).

The damping fluid is standard hydraulic fluid, and the system is bled after filling to remove all traces of air.

4.4.2 Calibration

Calibration of the static test rig is performed with the aid of an extension arm with a series of weights hung at its end. The arm effectively amplifies the force which is applied at the deflection bar by a factor determined by the arm length (as is illustrated in figure 4.8).

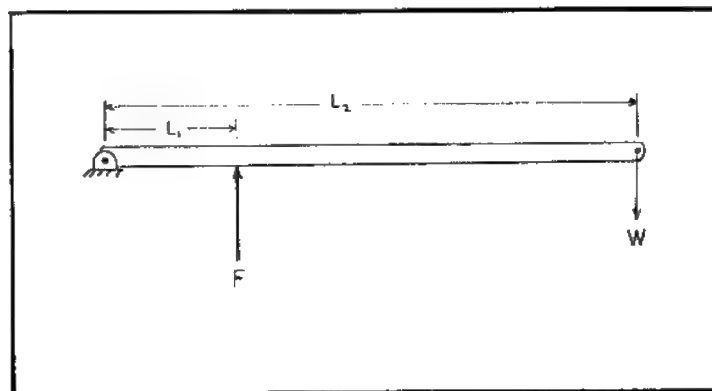


Figure 4.8. Extension arm setup for calibration of the test rig.

The force applied at the deflection bar F , is given by:

$$F = \frac{L_2}{L_1} \frac{W_a}{2} + W \quad (4.5)$$

where W is the total weight hung at the bar end, W_a is the arm weight. Since W_a , L_1 , L_2 are constant for a given arm, this can be written as:

$$F = C_1 W + C_2 \quad (4.6)$$

where $C_1 = L_2 / L_1$ and $C_2 = C_1 W_a / 2$.

Calibration is carried out by hanging a series of weights and recording the output voltage. The calibration data is presented in the form of voltage V , as a function of force, having fitted the points through a second order curve (through the origin) to yield a calibration curve of the following form:

$$F = C_3 V + C_4 V^2 \quad (4.7)$$

where C_3 and C_4 are constants.

CHAPTER 5: RESULTS (Theoretical and Actual)

5.1.1 Nozzle Testing

The thrust-time curves for the conical and cubic nozzle are presented in figure (5.1) and (5.2), respectively. The time period between successive points is 8.6 milliseconds (i.e. every 5th point is printed). For comparison purposes, both results are presented on a single curve illustrated in figure (5.3) (note: different time base).

A summary of the performance for both of these nozzles is presented in Table 5.1.

	Conical	Cubic
Total impulse (N-s)	288	281
Specific impulse (s)	130.5	127.3
Max. thrust (N)	1155	1075
Thrust duration (s)	0.38	0.40
Propellant mass (kg)	0.225	0.226

Table 5.1 Comparison of conical and cubic nozzle motor tests.

Both tests were conducted under similar environmental conditions, although on separate dates.

It should be noted that neither nozzle suffered any erosion or other detrimental effects after firings. This has been found to be the case even after multiple firings. (> 10) making the motors indefinitely reusable.

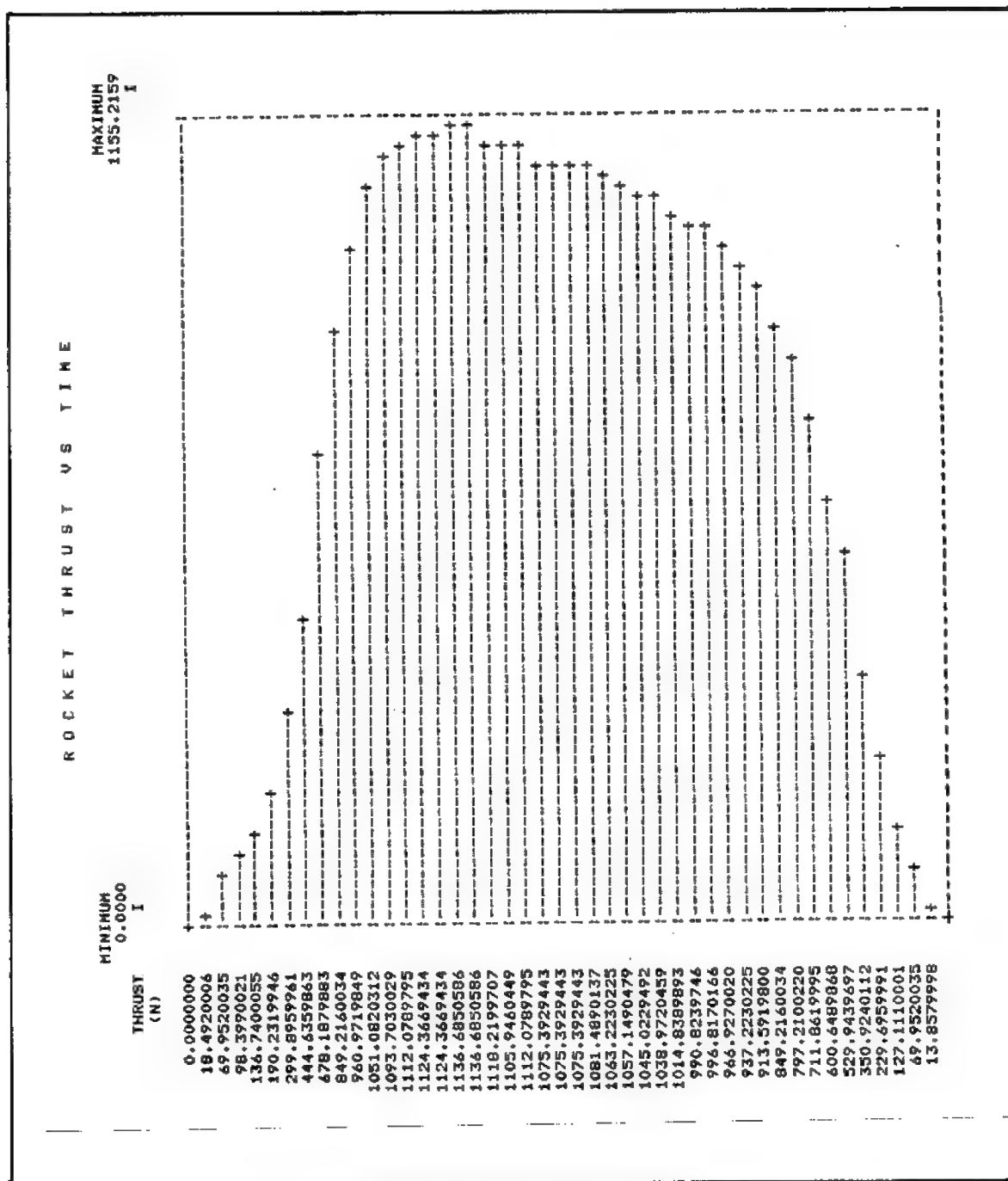


Figure 5.1. Actual thrust-time curve for the conical nozzle motor test.

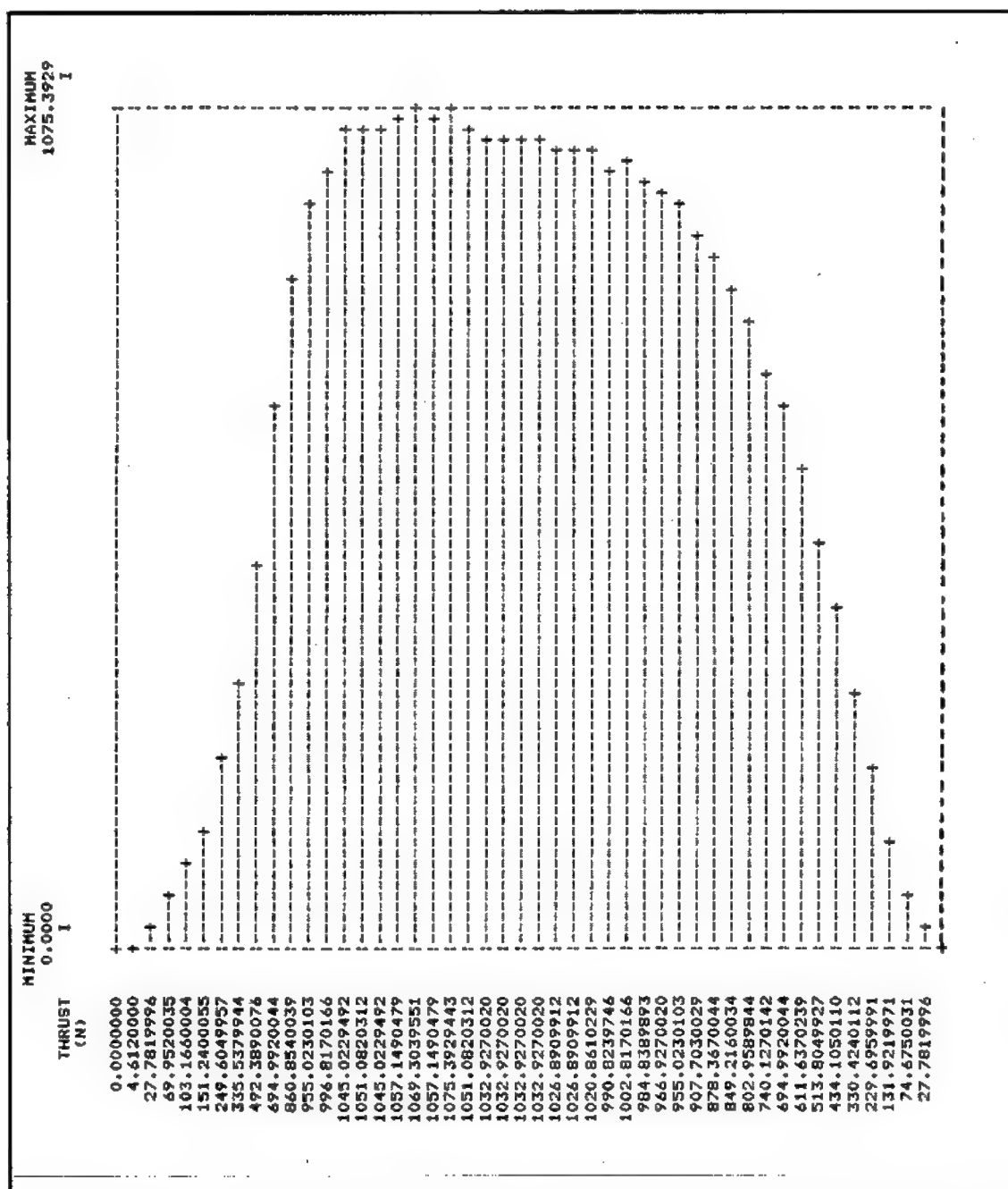


Figure 5.2. Actual thrust-time curve for the cubic nozzle motor test.

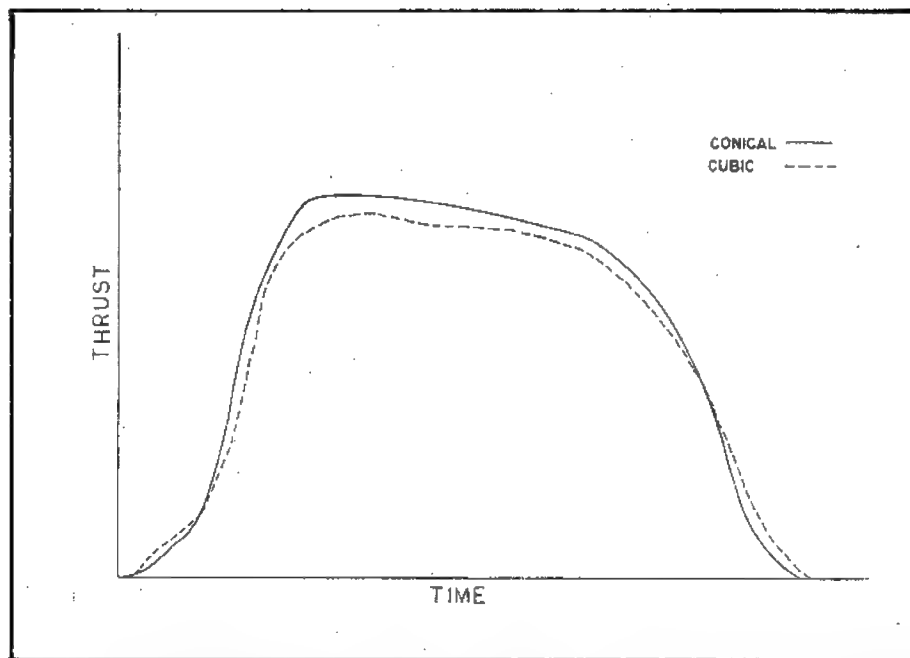


Figure 5.3. Comparison of the conical and cubic nozzle motor thrust-time curves.

5.1.2 Motor Testing of Varying O/F Ratio Grains.

The results of this series of tests are presented in Table 5.2. Only the values for the average specific impulse are presented because it is felt that this would provide the most useful base for comparison. It should be noted that a non-standard conical nozzle was used for this series of tests, resulting in lower overall values of specific impulse as compared to the standard conical nozzle as described earlier. Therefore, the results should be used for comparison purposes only.

The impulse for the 75/25 O/F ratio was not measurable due

to poor burnings resulting from the high oxidizer percentage. A large quantity of liquid matter (potassium carbonate) was ejected from the nozzle, and burning continued for about half a minute.

O/F Ratio	$\bar{I}_{sp}$ (s)
50/50	49.9
55/45	93.6
60/40	100
65/35	106
70/30	106
75/25	---

Table 5.2 Comparison of varying O/F ratio motor tests.

The 50/50 O/F ratio grain also resulted in poor firings. Large volumes of carbon were ejected with much remaining in the combustion chamber after firing in the form of a single porous mass.

Each of these test firings were recorded on 8mm colour movie film for post firing examination of the test. The results showed that the exhaust flame varied in size and colour with differing O/F ratios. For the more fuel rich ratios, the flame was smaller and more orange in colour. For the more oxidizer rich ratios, the flame increased in size with a more purple coloured hue, a result of the potassium compound in the exhaust. As well, it was found that the nozzle throat became hotter with increasing oxidizer percentage, with the exception of the 75/25 O/F ratio. The throat colour ranged from a dull red (50/50 O/F

ratio) to a bright yellow (70/30 O/F ratio). This result is consistent with the theoretical prediction for combustion temperature as a function of O/F ratio as discussed earlier.

A comparison of the propellant properties T_0 , M' , CV and X for varying O/F ratios as predicted by theory is shown in figure (5.4).

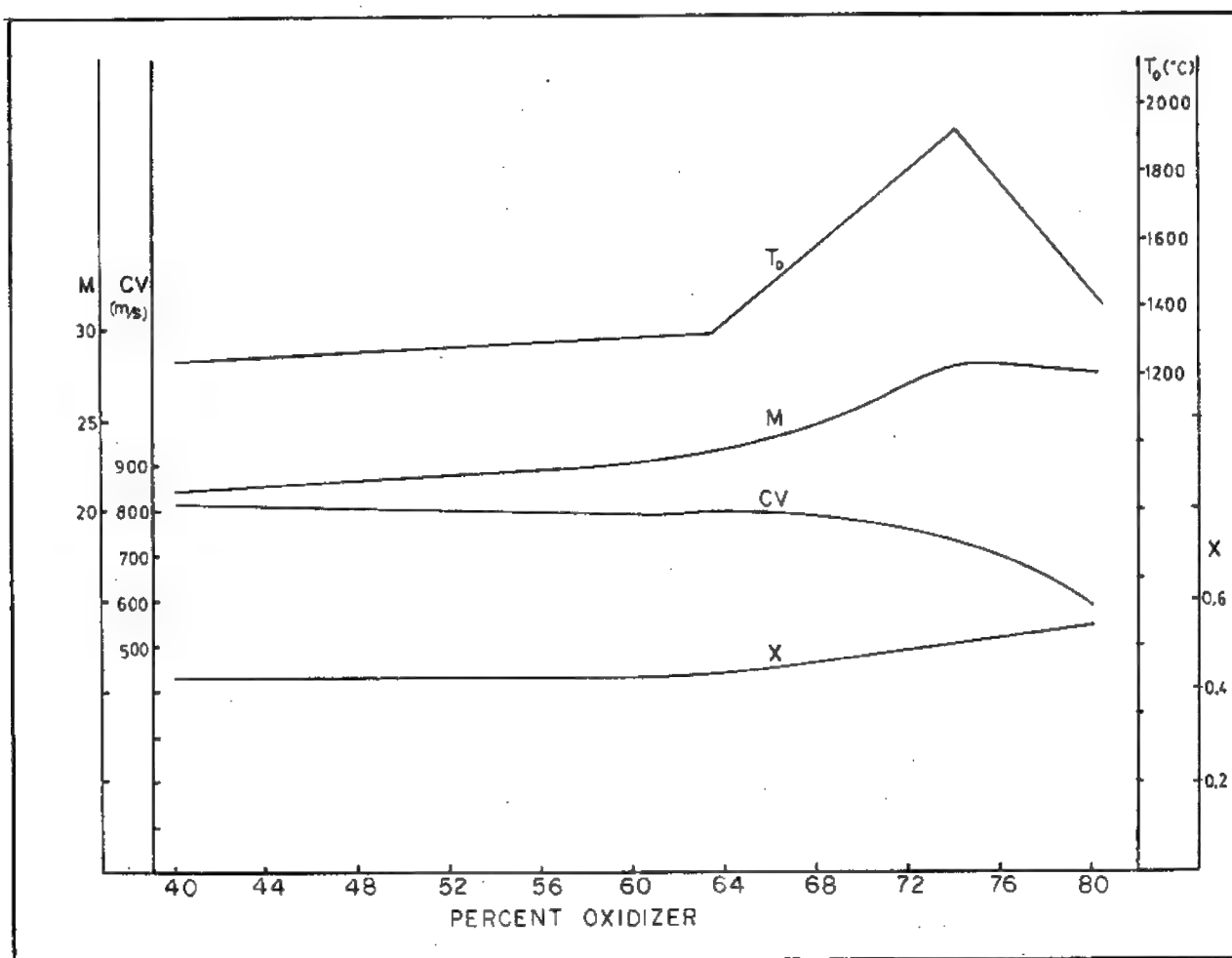


Figure 5.4. Comparison of theoretical properties:
 (1) Combustion temperature, T_0
 (2) Average molecular weight of exhaust gases, M'
 (3) Characteristic velocity, CV
 (4) Particle mass fraction, X.

5.1.3 Burnrate Testing

The results of burnrate testing for various O/F ratios are presented in figure (5.5). The conditions of testing were atmospheric pressure and room temperature. The burnrate results for the 75/25 O/F ratio are not shown due to the erratic burnings encountered.

As mentioned earlier, burnrate testing has not yet been conducted for conditions of elevated pressures to determine the burnrate-pressure relationship. However, it is possible to est-

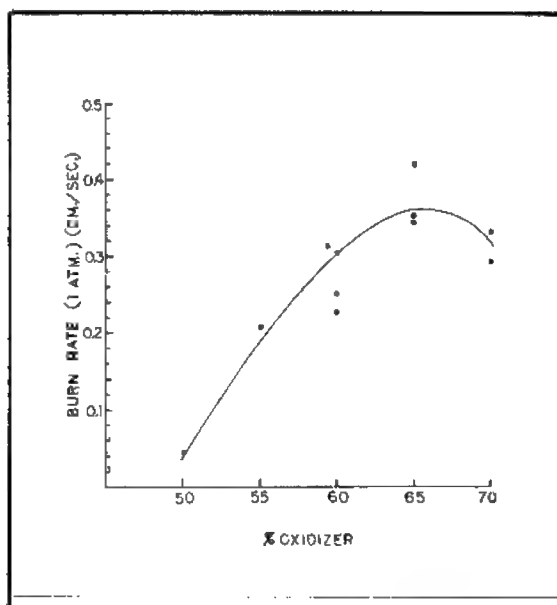


Figure 5.5. Comparison of atmospheric burnrate testing for varying O/F ratios.

imate the pressure dependence in order to determine an approximate value for both the coefficients a and n (from equation 2.10), using the results of the motor testing. This method involves

estimating the time required for the grain web (wall thickness) to burn through under motor operating conditions. The validity of this method lies in the assumption that essentially all the surface regression occurs at high pressure. This pressure, taken as P_o , occurs during the steady-state operation of the motor, allowing an estimation of this time period from the thrust-time curve (see figure 5.6).

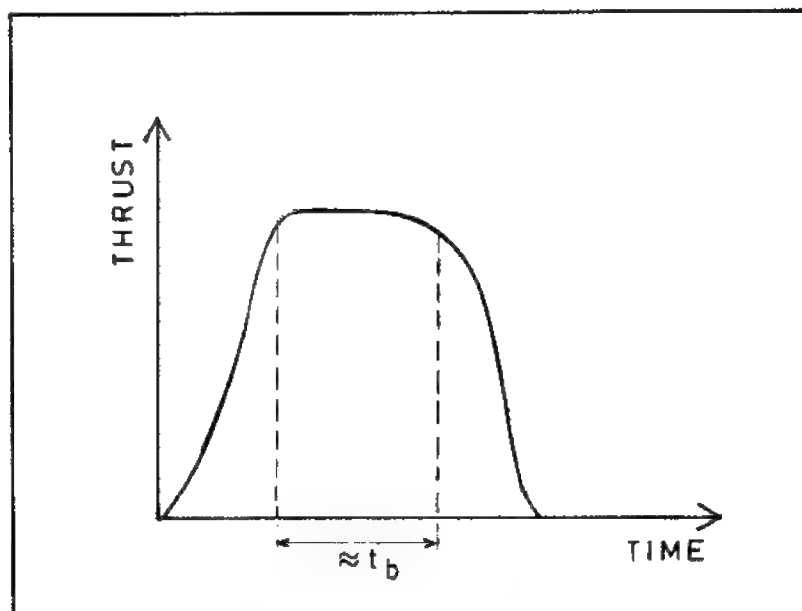


Figure 5.6. Estimation of web burnthrough time from actual thrust-time curve.

Since burning progresses from both the inside and outside surfaces of the grain, the web expansion rate is effectively doubled. The burnrate at a pressure of P_o can then be expressed as:

$$\left. r \right|_{P=P_o} = \frac{R_o - R_i}{2 t_w} = a P_o^n \quad (5.1)$$

where R_o and R_i are the grain outside and inside radii.

Combining this equation with the results of the atmospheric burnrate testins:

$$r \Big|_{P=P_{atm}} = a P_{atm}^n \quad (5.2)$$

and the steady-state pressure equation (3.34), results in a set of three simultaneous nonlinear equations in a , n , and P_o .

At a pressure of one atmosphere, the burnrate is taken as $r = 0.335$ cm/s (65/35 O/F ratio). Utilizing the thrust-time curve as shown in figure (5.1), the burnrate at P_o can be approximated at 2.22 cm/s. Equation (3.34) is applied with the following values shown below:

$$\begin{aligned} \rho_p &= 1.83 \text{ g/cm}^3 \\ A_b &= 280 \text{ cm}^2 \\ A^* &= 0.6825 \text{ cm}^2 \\ k &= 1.283 \\ T &= 1620 \text{ K} \end{aligned}$$

The value for A_b stated above is 10 percent greater than the value calculated by equation (2.13). This is an estimation of the actual surface area with surface flaws taken into account. Solutions of equations (5.1), (5.2) and (3.34) with the above values gives the following results for a , n and P_o :

$$\begin{aligned} a &= 0.85 \\ n &= 0.407 \\ P_o &= 10.5 \text{ MPa. (1530 psia)} \end{aligned}$$

When the coefficients above are inserted into the expression for burnrate (equation 2.10), where P_o is expressed in MPa., the resulting r is expressed in cm/s.

5.1.4 Performance Parameters

The theoretical performance parameters of the motor based upon the two particle flow models, as well as the actual results are presented in Table 5.2. These results are based upon the motor equipped with the standard conical nozzle and the 65/35 O/F ratio propellant.

PARAMETER	THEORETICAL PARTICLE MODEL		ACTUAL VALUE
	$T_s \approx T_g$	$V_s \approx V_g$	
THRUST (MAX), N.	1697	1153	1155
THRUST TIME, SEC.	—	—	0.378
V_e , m/s.	1490	1821	—
T_g , °C	1406	1406	1347
P_0 , MPa	10.55	10.55	—
C_f	2.36	2.00	—
I , N-s.	408	226	288
I_{sp} , sec.	152	103	131*
λ	0.989	0.989	—
ξ_d	0.998	0.998	—
ξ_v	—	—	0.633
e	—	—	0.401
			* average

Table 5.2. Comparison of theoretical performance parameters to actual performance parameters.

The energy conversion efficiency is based upon the actual performance of the motor as compared to the maximum performance possible, which would be attained for the first particle flow model.

CHAPTER 6: DISCUSSION OF RESULTS

6.1 Nozzle Testing

Contrary to expectations, the performance of the cubic nozzle was somewhat lesser to that of the conical nozzle, delivering a specific impulse 2.5 percent lower. This result however, is tentative for only a single firing of the cubic nozzle was performed due to time limitations. Clearly, however, no significant increase in nozzle efficiency is implied. This suggests that the conical nozzle is certainly a satisfactory design. Considering the far greater simplicity of fabrication, the conical design would have to be considered the superior choice for the amateur rocket enthusiast.

If further testing reinforces the present findings, the lesser performance of the cubic nozzle could be a result of its greater maximum divergence angle (20 degrees v.s. 12) and/or its shorter length (17% shorter). The latter of these would be consistent with the particle flow theory where a decrease in performance would be a direct result of particle lag. In a shorter nozzle the particles would have less time to accelerate to the gas velocity before exiting the nozzle. This would suggest that for a small motor with a high mass fraction of particles, the nozzle should have a small divergence angle in order to increase the length. The existing divergence angle should possibly be reduced to 10 or 8 degrees. As well, the effects of reducing the convergence angle in order to increase the overall length of the nozzle should be investigated. Possibly the best approach to the

design of a nozzle for such conditions would be to shape the contour in a manner such that the maximum flow acceleration is reduced, to minimize particle lag.

6.2 Motor Testing of Varying O/F Ratio Grains

Maximum performance for the hollow cylindrical grain is achieved with a moderately fuel rich O/F ratio. Testing indicates that the specific impulse is highest in the range of 65 to 70 percent oxidizer, tapering off with a lower oxidizer percentage, and being drastically lowered at high oxidizer ratios. The sensitivity at high oxidizer ratios is shown by the results that a maximum impulse is achieved at 70 percent oxidizer, but by increasing the ratio by 5 percent results in an immeasurable impulse.

The highly fuel rich ratios (O/F < 55/45) also resulted in greatly reduced performance, despite theoretical predictions that the specific impulse would remain essentially constant over the entire range from 40 to 70 percent oxidizer. This could be attributed to the observation that a large fraction of the solid matter (mostly carbon) remained in the combustion chamber in the form of a solid porous mass. This is precisely the conditions of the second particle flow model where $v \approx \text{constant}$, resulting in a specific impulse reduced by a factor of $(1-X)$.

At very high O/F ratios (O/F > 70/30), the reduction in performance could as well be explained by this particle flow model. Examination of the exhaust blast area revealed that the potassium carbonate particles ejected from the nozzle were actually in the

form of sizeable droplets. This would create the conditions of $v \approx \text{constant}$, where the relatively large particle size would not allow acceleration of the particles to a high velocity. The formation of the droplets could be explained thusly: during combustion a voluminous quantity of potassium carbonate is formed, being in the liquid state. As flow occurs through the nozzle multiple collisions occur between the liquid particles, resulting in cohesion and effective growing in particle size until either the maximum stable size is achieved, or exit occurs from the nozzle.

The practical range, therefore, for O/F ratio selection appears to be an oxidizer percentage of between 55 and 70 percent. Coupling this result with the realization that actual combustion temperatures appear to parallel theoretical, would indicate that since lower combustion temperatures might be more desirable, the lower O/F ratios might be more suitable. If however, the combustion temperatures encountered at the upper end of the O/F ratio pose no practical problem, the range for best performance could be reduced to the 65 to 70 percent oxidizer range. Since no erosion or other problems have been encountered due to these temperatures, this would appear to be the optimal range. One final factor to be considered is the castability of the grain. Since a more sucrose rich propellant is easier to cast, the best overall ratio would then appear to be the existing 65/35 O/F ratio. It should be noted that since testing occurred in steps of 5 percentage points, that the optimum ratio is therefore within a certain plus-minus range of this value.

6.3 Burnrate Testing

The results of testing indicate that burnrate is quite sensitive to the O/F ratio, with the maximum occurring at the 65/35 O/F ratio. For small rocket motors, a greater rate of burning is desirable to develop the high chamber pressures that such motors can readily operate at. With lower burnrates, it would be necessary to increase the grain surface (burning) area to compensate for the reduced rate of burning in order to maintain the gas generation rate. A decrease in throat area would as well be a solution to developing the desired pressure, however, a corresponding thrust loss would result.

Increased grain surface area can be achieved by modifying the geometry of the central bore, by moulding a star or other shape as discussed in section 2.2.3. This result can also be achieved by increasing the length/diameter ratio of the grain. However, both methods complicate the grain casting procedure and increase the likelihood of grain casting flaws. Therefore, practical considerations make the higher burnrate that can be achieved with the 65/35 O/F ratio a desirable trait.

The method of estimating the burnrate-pressure relationship appears to be satisfactory for a first approximation. However, it is difficult to gauge the accuracy at this point. Knowledge of the actual chamber pressure would certainly increase the precision of this technique. Due to the nonlinearity of the equations, the value for the chamber pressure, however, could be subject to appreciable error.

6.4 Performance Parameters

From table 5.2 , it can be seen that there is an appreciable difference in ideal thrust for the two particle flow models. In the second case where the particles are assumed to have no appreciable velocity, the thrust is 32 percent lower than the first case where all the particles are assumed to have a velocity equal to that of the gas flow. As a result of this rather significant variance, the choice of model will importantly affect the expected performance of a given motor design. Neither model would be expected to accurately represent the actual flow conditions since these are the two limiting cases. The actual particle flow conditions would require a consolidation of the two models, where the expected performance would lie somewhere between these two bounds. As well, the actual flow would be difficult to model since the real particle size would certainly be distributed, where the smaller particles might follow closely the gas flow velocity whereas the larger particles might have a significant velocity lag.

The actual thrust produced by the motor would therefore be expected to lie somewhere between these two ideal values, which it in fact does. The actual maximum thrust value (1155 N.) almost coincides with the ideal value for the second particle flow model (1153 N.), suggesting that the actual flow would be accurately described by this model. However, such a conclusion would be highly tentative, is not incorrect, since the calculated thrust values for these two models are almost directly proportional to chamber pressure. The chamber pressure used for these two calculations

(10.55 MPa.) is based on the value arrived at through the pressure-burnrate calculations described in the previous section which, as mentioned, could be subject to appreciable error. Any such error would be almost directly reflected in the ideal thrust computations. This factor renders the thrust parameter a poor basis of comparison of the actual to theoretical flow conditions. A far better comparison would be made utilizing the specific impulse, which is insensitive to chamber pressure variations.

The value of the actual average specific impulse (131 s.) is in fact close to the midpoint of the values predicted by the two particle flow models (152: first model; 103: second model). This implies that a consolidation of the two models would closely predict the behaviour of the actual flow.

The energy efficiency ϵ , and the velocity coefficient C_v , are employed to relate the actual thrust to the ideal thrust that would be achieved, as explained in section 3.2.7. The values obtained appear to be on the low side, again possibly as a result of error in the chamber pressure assumed.

The actual value for the thrust coefficient C_f , would also likely lie between the two values as shown. This result demonstrates that the thrust can be more than doubled with a properly designed nozzle, with the importance of the divergent section apparent.

The nozzle exit velocity v_e is shown to be significantly higher for the second particle flow model, where this represents the sas only value. The exit velocity for the first model where

the particles are assumed to have the same velocity as the gas flow is reduced as a result of accelerating the particles.

The actual combustion temperature (1347 C.) was found to be in close agreement with the theoretical value (1406 C.) being 4 percent lower. The slightly lower combustion temperature would be a result of incomplete combustion and nonadiabatic conditions.

The final parameters to be considered are the discharge coefficient C_d and the divergence correction λ . It can be seen that only minor thrust losses are a result of these two factors.

CHAPTER 7: CONCLUSIONS

Theoretical analysis of a small solid propellant rocket motor was conducted, with the performance results presented in the form of two limiting models. The actual findings were expected to lie within this range, and experimental measurements found this to be the case. This result demonstrated the importance of considering the effects of two-phase flow for small solid propellant rocket motors.

Theoretical analysis suggested that the propellant performance should remain essentially constant over a fairly wide range of oxidizer/fuel ratios, however, other factors such as burn-rate and castability reduced the acceptable range. The best overall ratio was found to be the presently employed ratio of 65/35.

Significant improvement of the motor's performance does not appear likely to be achieved by variation of the nozzle design. The conical nozzle emerged as a highly satisfactory design when the simplicity of fabrication is considered. Marginal increase in nozzle efficiency might be obtained by reducing the convergence and/or divergence angles.

A thrust-time curve of the rocket motor's performance was obtained, providing a basis upon which to determine the expected acceleration, velocity, and altitude of the rocket in actual flight.

RECOMMENDATIONS

The following recommendations are suggested to provide a more complete understanding of the rocket motor's and propellant's performance, and to provide a sound basis for the design of larger rocket motors:

1. Conduct testing in order to determine the actual chamber pressure during firing, possibly utilizing a pressure transducer system to obtain a pressure-time curve. As well, a method of determining burnrate as a function of chamber pressure should be investigated utilizing this data.

2. Other methods of determining the pressure-burnrate relationship should be considered, such as construction of a constant pressure vessel to study the rate of burning at elevated pressures.

3. Investigation into modifying the conical nozzle design in order to reduce particle acceleration might be attempted.

4. Further studying of two-phase flow in an attempt to consolidate the particle flow models would certainly prove useful.

As well, it is suggested that a computer program be developed to solve the equations of motion involved with a rocket vehicle in flight, utilizing the obtained thrust-time data in conjunction with vehicle drag data (either experimentally obtained or theoretically modeled).

NOMENCLATURE

a	burnrate coefficient
$\bar{a}$	acoustic velocity
A	area
A _b	burning area
A/D	analogue / digital
AFT	adiabatic flame temperature
*	
A*	critical throat area
c	effective exhaust velocity
C.	Celsius
C _f	Coefficient of thrust
C _p	Specific heat at constant pressure
C _s	Specific heat of solid (liquid)
C _v	Specific heat at constant volume
CV	characteristic velocity
D _i	grain inside (bore) diameter
D _o	grain outside diameter
e	energy conversion efficiency
f	mass fraction
F	thrust

g	acceleration of gravity
h	enthalpy
h_f°	enthalpy of formation
I	total impulse
I_{sp}	specific impulse
k	ratio of specific heats
K	Kelvin
L	grain length
$\dot{m}$	mass flowrate
$\dot{m}_g$	gas mass flowrate
$\dot{m}_s$	particle (solid) mass flowrate
m.p.	melting point
M	Mach number
M'	molecular weight
n	burnrate exponent
N	Newton
O/F	oxidizer/fuel
P	pressure
P_a	ambient pressure
P_e	nozzle exit pressure
P_o	stagnation pressure
r	burnrate
R	molar gas constant

R'	universal gas constant
t	time
t_b	burntime
T	temperature
T_o	stagnation temperature
v	velocity
v_e	nozzle exit velocity
v_g	gas velocity
v_s	particle (solid) velocity
V_c	chamber volume
X	particle (solid) mass fraction
α	divergence angle
ζ_d	discharge coefficient
ζ_v	velocity coefficient
λ	divergence correction factor
ρ	density
ρ_g	gas density
ρ_p	propellant density
ρ_o	stagnation density
μ	micrometre

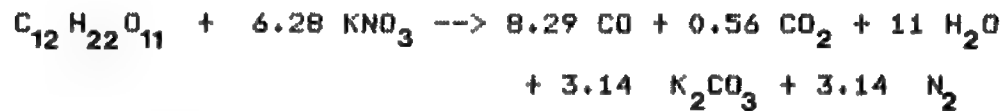
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APPENDIX A

Calculation of the Adiabatic Flame Temperature assuming the combustion equation shown below:



with the assumptions of no work, no changes in K.E. or P.E., and adiabatic conditions, the first law reduces to:

$$\sum_R n_i [\bar{h}_f^\circ + \Delta \bar{h}]_i = \sum_P n_e [\bar{h}_f^\circ + \Delta \bar{h}]_e$$

where the enthalpies of formation are given below:

Constituent	State	$\bar{h}_f^\circ$ (kJ/kmol)	Reference
$\text{C}_{12}\text{H}_{22}\text{O}_{11}$	solid	-2,222,100	[31]
KNO_3	solid	- 493,205	[32]
CO	gas	- 110,529	[33]
CO_2	gas	- 393,522	[33]
H_2O	gas	- 241,827	[33]
K_2CO_3	liquid	-1,146,835	[32]
N_2	gas	0	----

The values for Δh are zero for the reactants. For the products, the values for Δh are available from standard enthalpy tables (except for K_2CO_3) such as from reference [33]. The value for K_2CO_3 is obtained from the expression:

$$\Delta \bar{h} = \int_{T_1}^{T_2} \bar{C}_P dT + \bar{h}_{tr}$$

where $\bar{C}_P = 0.0948 T + 94.25$ kJ/kmol-K [32].

and $\Delta \bar{h}_{tr} = 27,633$ kJ/kmol [32].

This leads to, for K_2CO_3 :

$$\Delta \bar{h} = \left[(0.0474 T^2 + 94.25 T) - 32,296 \right] + 27,633 \text{ kJ/kmol}$$

Insertion of above values and the expression for enthalpy for the potassium carbonate gives the following expression:

$$\begin{aligned} 8.29 (\Delta \bar{h}_{CO}) + 0.564 (\Delta \bar{h}_{CO_2}) + 11 (\Delta \bar{h}_{H_2O}) + 3.14 (0.0474 T^2 \\ + 94.25 T) + 3.14 (\Delta \bar{h}_{N_2}) - 2,099,714 = 0 \end{aligned}$$

A solution for T (AFT) is obtained through a trial and error method by inserting values for enthalpies at the trial temperature.

Solution of the equation is obtained at $T = 1681$ K.

APPENDIX B

Determination of the average value for the ratio of specific heats k , for the 65/35 O/F ratio for flow through the nozzle.

The assumption is made where: $P = 10.55 \text{ MPa}$,
 D

$P = 0.101 \text{ MPa}$,
 E

$T = 1629 \text{ K}$,
 D

Use of the expression:

$$C = \frac{n_i}{n_t} C_{P_i} + \frac{n_j}{n_t} C_{P_j} + \frac{n_k}{n_t} C_{P_k} + \dots$$

and the expression for C as a function of temperature given below [3],

$$\text{CO: } C_P = 69.145 - 0.7046 \theta^{0.75} - 200.77 \theta^{0.5} + 176.76 \theta^{-0.75}$$

$$\text{CO}_2: C_P = -3.736 + 30.529 \theta^{0.5} - 4.1034 \theta + 0.024198 \theta^2$$

$$\text{H}_2\text{O: } C_P = 143.05 - 183.54 \theta^{0.25} + 82.751 \theta^{0.5} - 3.6989 \theta$$

$$\text{N}_2: C_P = 39.060 - 512.79 \theta^{-1.5} + 1072.7 \theta^{-2} - 820.4 \theta^{-3}$$

where $\theta = \frac{T}{100}$, valid in the range of 300 - 3500 K

k can be expressed as $k = \frac{C_P}{C_P - R'}$, where R' is the universal gas constant.

The above expression for C becomes:
 P

$$C_P = \frac{1}{23} (8.29 C_{P_{\text{CO}}} + 0.564 C_{P_{\text{CO}_2}} + 11 C_{P_{\text{H}_2\text{O}}} + 3.14 C_{P_{\text{N}_2}})$$

The use of the isentropic relation:

$$T_e = T_o \left(\frac{P_e}{P_o} \right)^{\frac{k-1}{k}}$$

allows the temperature of the gas flow at nozzle exit to be determined. The specific heat is calculated at T_o and T_e and an average value determined:

$$\text{avg. } C_p = (C_{p,T_o} + C_{p,T_e}) / 2$$

whereas the average value for k can be found from the expression relating C_p to k . However, since it is the average value of k that is to be used in the isentropic expression to obtain a true value for T_e , successive iteration is required to obtain the desired average value for k for flow through the nozzle:

T_o	T_e	avg. C_p	avg. k
1620	---	41.97	1.247
1620	646	38.05	1.280
1620	588	37.76	1.282
1620	583	37.74	1.283
1620	583	-----	-----

The value of T_e has reached a constant value, so the value for average k for flow through the nozzle is:

$$k = 1.283$$

APPENDIX C

Derivation of the expression for v for two-phase flow for the case where $T \approx T_s$ and $v \approx v_s$.

Consider a frozen flow model where particles are formed within the combustion chamber with no subsequent growth or formation. Under such conditions, $\dot{m}_s$ and $\dot{m}_g$ are constant throughout the nozzle. Applying the general momentum relation to an incremental control volume gives the result:

$$-A dP = \dot{m}_s (v_s + dv_s) + \dot{m}_g (v_g + dv_g) - \dot{m}_s v_s - \dot{m}_g v_g \quad (1)$$

where A is the cross sectional area at a given point. From continuity:

$$\dot{m}_s = \rho_s A v_s \quad \text{and} \quad \dot{m}_g = \rho_g A v_g \quad (2)$$

Combining these two equations we obtain:

$$-dP = \rho_s v_s dv_s + \rho_g v_g dv_g \quad (3)$$

The energy equation for steady isentropic flow relates the particle and gaseous enthalpies and kinetic energies according to:

$$\dot{m}_s (C_{ps} dT_s + v_s dv_s) + \dot{m}_g (C_{pg} dT_g + v_g dv_g) = 0 \quad (4)$$

Using the mass fraction of particles X , this equation can be re-written as:

$$X C_{ps} dT_s + (1-X) C_{pg} dT_g + X v_s dv_s + (1-X) v_g dv_g = 0 \quad (5)$$

From equation (3) we obtain:

$$v \frac{dv}{ds} = - \frac{dP}{\rho_g} - \frac{\rho_s}{\rho_g} v \frac{dv}{ds} = - \frac{dP}{\rho_g} - \frac{X}{1-X} v \frac{dv}{ds} \quad (6)$$

Substitution of this expression into (5) gives:

$$\frac{X}{1-X} C_s \frac{dT}{ds} + \frac{C_p}{P} \frac{dT}{ds} + \frac{X}{1-X} (v - v_s) \frac{dv}{ds} = - \frac{dP}{\rho_g} \quad (7)$$

The solution of this equation requires knowledge of the heat transfer and drag processes in order to express T and v in terms of the overall flow properties. However, if the heat transfer is considered to be very fast ($T \approx T_s$), and particle drag very high ($v \approx v_s$), then a limiting solution can be obtained.

The equation would then reduce to:

$$\left(\frac{X}{1-X} C_s + C_p \right) \frac{dT}{ds} = - \frac{dP}{\rho_g} \quad (8)$$

assuming the ideal gas relation, $\rho_g = P/RT$, this becomes:

$$\left\{ \frac{[X(1-X)] C_s + C_p}{R} \right\} \frac{dT}{T} = - \frac{dP}{P} \quad (9)$$

Integration from stagnation conditions to exit conditions gives:

$$\frac{T_e}{T_0} = \left(\frac{P_e}{P_0} \right)^m \quad \text{where } m = \frac{R}{[X(1-X)] C_s + C_p}$$

For these conditions, equation (5) becomes:

$$\left[X C_s + (1-X) C_p \right] dT + v dv = 0$$

which may be integrated to give v_e in terms of T_e and T_o , and where
 $C_p = k R / (k-1)$;

$$v_e = \sqrt{2 T_o \left[X C_s + (1-X) \frac{k R'}{(k-1) M'} \right] \left[1 - \left(\frac{P_e}{P_o} \right)^m \right]}$$

where

$$m = \left[\frac{X C M'}{(1-X) R'} + \frac{k}{k-1} \right]^{-1}$$

APPENDIX D

Derivation for the expression for thrust F , for two phase flow where $T \approx T_s$ and $v \approx v_s$.

The expression for gas mass flow rate through a nozzle is given by the expression [34]:

$$\dot{m}_s = \frac{A^* P_o}{\sqrt{R T_o}} \sqrt{k \left(\frac{2}{k+1} \right)^{\frac{k+1}{k-1}}}$$

For the case where all particles are accelerated through the nozzle (none remaining in combustion chamber), the expression for total mass flowrate $\dot{m}$, is given by:

$$\dot{m} = \dot{m}_s + \dot{m}_p$$

since $\frac{\dot{m}_s}{\dot{m}} = \frac{X}{1-X}$, this would give

$$\dot{m} = \dot{m}_s + \dot{m}_p \frac{X}{1-X} = \dot{m}_s \frac{1}{1-X}$$

Equation (1) can be used to express the total mass flowrate in terms of the gas where:

$$\dot{m} = \frac{1}{1-X} \frac{A^* P_o}{\sqrt{R T_o}} \sqrt{k \left(\frac{2}{k+1} \right)^{\frac{k+1}{k-1}}}$$

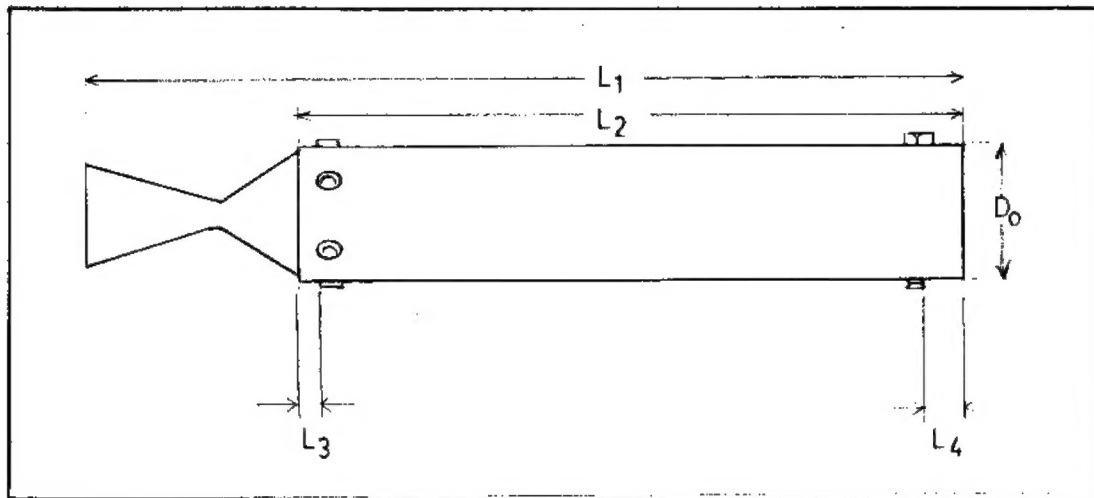
Since $F = m v_e + (P_e - P_a) A_e$, using the previously derived expression for v_e , (appendix C), we can write:

$$F = A_e^* P_o \frac{1}{1-X} \sqrt{\frac{2 k M'}{R'} \left(\frac{2}{k+1} \right)^{\frac{k+1}{k-1}} \left[X C_s + \frac{(1-X) k R'}{(k-1) M'} \right] \left[1 - \left(\frac{P_e}{P_o} \right)^m \right]}$$

where $m = \left[\frac{X C M'}{(1-X) R'} + \frac{k}{k-1} \right]^{-1}$

APPENDIX E

Rocket Motor Design and Specifications



$$L_1 = 24.6 \text{ cm.}$$

$$L_3 = 0.5 \text{ cm.}$$

$$D_0 = 3.795 \text{ cm.}$$

$$L_2 = 18.6 \text{ cm.}$$

$$L_4 = 0.65 \text{ cm.}$$

$$D_i = 3.495 \text{ cm.}$$

Chamber wall thickness, $t = 1.5 \text{ mm}$

Chamber wall material - mild steel tubing, $s_u = 400 \text{ MPa.}$

Safety Bolt - 3/16 in. x 1 1/2 in.; grade 6

tensile strength, $s_u = 900 \text{ MPa. (rated)}$

shear strength, $s_s = 448 \text{ MPa. (tested)}$

Nominal chamber pressure required to sever safety bolt, P'

$$P' = 2 s_s \left(\frac{D_b}{D_i} \right)^2 = 17 \text{ MPa.}$$

where D_b is bolt diameter.

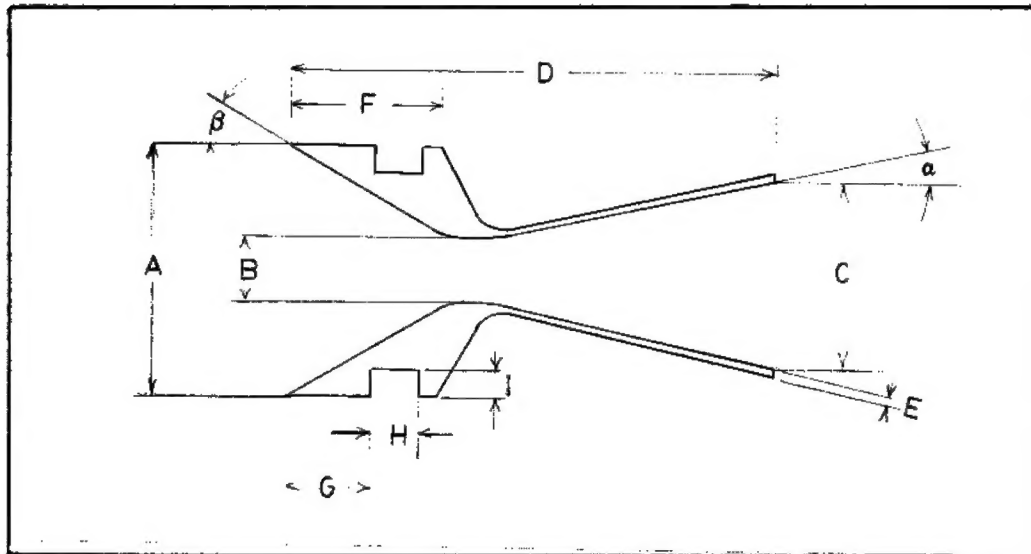
Nominal chamber pressure required to burst case, P''

$$P'' = \frac{2 S_u t}{D_i} = 35 \text{ MPa.}$$

where S_u is chamber tensile strength.

Nominal chamber operating pressure = 10.55 MPa.

Conical Nozzle Design



Dimensions in cm. :

A	3.50	E	0.08	I	0.35
B	0.932	F	2.2	a	12°
C	2.68	G	1.2	β	30°
D	6.8	H	0.70		

Expansion Ratio: 8.27 : 1

Material: mild steel

Inside flow surface: polished

APPENDIX F

STRAIN GAUGE BRIDGE - AMPLIFIER CIRCUIT

